CMSC 303 Introduction to Theory of Computing, VCU
Fall 2017, Assignment 3
Due: Thursday, September 28, 2017 at start of class

Total marks: 71 marks + 7 bonus marks for LaTeX.

Unless otherwise noted, the alphabet for all questions below is assumed to be $\Sigma = \{0, 1\}$.

1. [12 marks] This question develops your ability to devise regular expressions, given an explicit definition of a language. For each of the following languages, prove they are regular by giving a regular expression which describes them. Justify your answers.
   (a) $L = \{x \mid x$ begins with a 0 and ends with a $1\}$.
   (b) $L = \{x \mid x$ contains at least four 0's$\}$.
   (c) $L = \{a, aa, \epsilon\}$.
   (d) $L = \{x \mid$ the length of x is at most $4\}$.
   (e) $L = \{x \mid x$ doesn't contain the substring $110\}$.
   (f) $L = \{x \mid |x| > 0$, i.e. x is non-empty$\}$.
2. This question tests your understanding of how to translate a regular expression into a finite automaton. Using the construction of Lemma 1.55, construct NFAs recognizing the languages described by the following regular expressions.
   (a) [5 marks] $R = \emptyset^*$.
   (b) [15 marks] $R = (0 \cup 1)^* 111 (0 \cup 1)^*$.
3. [15 marks] This question tests your understanding of how to translate a finite automaton into a regular expression. Consider DFA $M = (Q, \Sigma, \delta, q, F)$ such that $Q = \{q_1, q_2, q_3\}$, $q = q_1$, $F = \{q_1, q_3\}$, and δ is given by:

δ	0	1
q_1	q_2	q_2
q_2	q_2	q_3
q_3	q_1	q_2

   Draw the state diagram for M, and then apply the construction of Lemma 1.60 to obtain a regular expression describing $L(M)$.
4. [10 marks] This question allows you to practice proving a language is non-regular via the Pumping Lemma. Using the Pumping Lemma (Theorem 1.70), give formal proofs that the following languages are *not* regular:
   (a) $L = \{www \mid w \in \{0, 1\}^*\}$.
   (b) $L = \{x \mid x \in \{0, 1\}^*$ is not a palindrome$\}$. Recall a palindrome is a string that looks the same forwards and backwards. Examples of palindromes are "madam" and "racecar".

5. This question further tests your understanding of the subtleties of the Pumping Lemma:

(a) [5 marks] Let $B_1 = \left\{0^k 1 x 0^k \mid k \geq 1 \text{ and } x \in \Sigma^*\right\}$. Give a formal proof that B_1 is not a regular language.

(b) [7 marks] Let $B_2 = \left\{0^k x 0^k \mid k \geq 1 \text{ and } x \in \Sigma^*\right\}$. Show that B_2 is a regular language.

(c) We just proved in part (b) above that B_2 is regular. Consider now the following argument, which claims to prove that B_2 is, in fact, not regular.

Assume B_2 is regular, and let p be the pumping length. Consider string $s = 0^p 1 0^p \in B_2$, and decompose it as $s = xyz$ with $x = \epsilon$, $y = 0^p$, $z = 10^p$. Then, pumping s down by setting $i = 0$ yields string $s' = xy^i z = xy^0 z = 10^p \notin B_2$. Hence, by the Pumping Lemma, we have a contradiction. We conclude that B_2 is not regular.

i. [2 marks] What is wrong with this proof?