Introduction to Mathematical Reason	MATH 300 Test #2	April 10, 2012
Name:____________________	R. Hammack	Score:______

1. (14 points) Suppose A, B and C are sets, and $C \neq \emptyset$. Prove that $A \times C \subseteq B \times C$ if and only if $A \subseteq B$.

2. Suppose A, B, C and D are sets.

(a) (10 points) Prove that $(A \times B) \cup (C \times D) \subseteq (A \cup C) \times (B \cup D)$.

(b) (10 points) Give a counterexample showing that it is not always true that $(A \times B) \cup (C \times D) \ = \ (A \cup C) \times (B \cup D)$.

3. (10 points) Draw diagrams for all the different relations on $A = \{a, b, c\}$ that are both reflexive and symmetric, but **not** transitive.

4. (14 points) Prove that $3^1 + 3^2 + 3^3 + \cdots + 3^n = \dfrac{3^{n+1} - 3}{2}$ for every $n \in \mathbb{N}$.

5. (14 points) Recall that the Fibonacci Sequence is defined as $F_1 = 1$, $F_2 = 1$ and $F_n = F_{n-1} + F_{n-2}$ for $n \geq 3$.

Use induction to prove that $\sum_{i=1}^{n} F_i^2 = F_n F_{n+1}$ for every $n \in \mathbb{N}$.

6. (14 points) Prove or disprove:
   If A and B are sets, then $\mathscr{P}(A) - \mathscr{P}(B) \subseteq \mathscr{P}(A - B)$.

7. (14 points) Prove or disprove:
   Suppose R and S are equivalence relations on a set A. Then $R \cup S$ is also an equivalence relation on A.