Introduction to Mathematical Reason	Test 1 MATH 300	March 8, 2011
Name: ____________	R. Hammack	Score: ______

Directions: Please answer the questions in the space provided. To get full credit you must show all of your work. Use of calculators and other electronic devices is not allowed on this test.

1. **Short answer.** Write each of the following sets by listing its elements between braces or describing it with a familiar symbol or symbols.

   (a) $\{n \in \mathbb{Z} : n^2 \leq 4\} =$

   (b) $\{x : (x,x) \in \mathbb{Z} \times \mathbb{N}\} =$

   (c) $\displaystyle\bigcap_{X \in \mathscr{P}(\mathbb{N})} \big(X \cup \{0\}\big) =$

   (d) $\mathscr{P}(\{1,2\}) - \mathscr{P}(\{2\}) =$

   (e) $\{X \subseteq \mathbb{R} : |X \cup \{0,1\}| = 2\} =$

2. **Short answer.** Write the following sets in set-builder notation.

   (a) $\{5,\ 8,\ 11,\ 14,\ 17,\ 20, \ldots\} =$

   (b) $\left\{\ldots, \frac{1}{16}, \frac{1}{8}, \frac{1}{4}, \frac{1}{2}, 1, 2, 4, 8, 16, \ldots\right\} =$

   (c) $\{\ldots,\ (-3,-4),\ (-2,-2),\ (-1,0),\ (0,2),\ (1,4),\ (2,6),\ (3,8),\ \ldots\} =$

3. Are $P \Rightarrow Q$ and $\sim(P \wedge \sim Q)$ logically equivalent? Support your answer with a truth table.

4. This problem concerns the following statement.
$P:$ Given any subset X of $\mathbb{R}$, there exists a subset Y of $\mathbb{R}$ for which $X \cap Y = \emptyset$ and $X \cup Y = \mathbb{R}$.

(a) Is the statement P true or false? **Explain**.

(b) Form the negation $\sim P$. Write your answer as an English sentence.
(The sentence may use mathematical symbols.)

5. This question concerns lists **of length 5** using the symbols A, B, C, D, E, F, G, H, **without repetition**.

   (a) How many such lists are possible?

   (b) How many such lists are there that are **not** in alphabetical order?

6. *Prove:* Suppose $a, b, c \in \mathbb{Z}$. If $a^2 \mid b$ and $b^3 \mid c$, then $a^6 \mid c$. (Hint: Try direct.)

7. *Prove:* Suppose $a \in \mathbb{Z}$. If a^3 is not divisible by 8, then a is odd. (Hint: Try contrapositive.)

8. *Prove:* If $a \in \mathbb{Z}$, then $4 \nmid (a^2 - 3)$. (Hint: Try contradiction.)

9. *Prove:* If $a, b \in \mathbb{Z}$ and $a + b$ is even, then $a^2 + b^2$ is even. (Hint: Try direct.)

10. *Suppose* $a, b, c \in \mathbb{Z}$, *and* $n \in \mathbb{N}$. *Prove the following:*
    If $a \equiv b \pmod{n}$ and $b^2 \equiv c^2 \pmod{n}$, then $a^2 \equiv c^2 \pmod{n}$.