Additional Counting Problems

1. Sections 6 and 7

   (a) A catered menu is to include soup, a main course, a dessert and a beverage. Suppose a customer can select from 2 soups, 3 main courses, 4 desserts and 3 beverages. How many different ways can the customer select the meal?

   (b) How many distinct four-digit nonnegative numbers (with possible leading zeros) are there?

   (c) How many distinct four-digit nonnegative numbers (with possible leading zeros) are there if at least two of the digits must be the same?

   (d) How many different sets of answers are possible on a test with 10 true-false questions?

   (e) How many different sets of answers are possible on a test with 10 multiple choice questions, each of which has 4 choices?

   (f) The call letters for radio stations in the United States consist of either 3 or 4 letters beginning with either a K or a W. How many different call letters are possible?

   (g) How many license plates are there consisting of two letters followed by four digits?

   (h) How many license plates are there consisting of two letters followed by four digits, if the digits cannot be repeated but the letters can?

   (i) In how many ways are there to arrange the letters A, B, C, D, and E, so that A and B are next to one another?

   (j) In how many ways are there to arrange the letters A, B, C, D, and E, so that A and B are not next to one another?

2. Sections 11 and 12

   (a) This problem concerns relations on the set $A = \{a, b\}$.

      i. Write out all the different relations on A. All together, there should be 16.

      ii. Which of the above relations on A are reflexive? (There should be 4 such relations.)

      iii. Which of the above relations on A are symmetric? (There are 8 such relations.)

      iv. Which relations on A are equivalence relations? (There are only 2.)

      v. How many different relations R on A are there that satisfy aRb?

   (b) This problem concerns relations on the set $A = \{a, b, c\}$. It is similar to the previous problem except that there are so many relations on A that you will not want to write them all down. Still, you can answer the questions using standard counting arguments.

      i. How many different relations on A are there?

      ii. How many different relations on A are there that are reflexive?

      iii. How many different relations on A are there that are symmetric?

      iv. How many different relations on A are there that are equivalence relations?

      v. How many different relations R on A are there that satisfy aRb?

3. Section 13

   (a) Assume that 26 books, consisting of 8 identical math texts, 6 identical french texts, and 9 identical art textss and 3 identical philosophy texts are lined up on a shelf. In how many different ways can this be done?

   (b) How many different anagrams of the following words can be made?

      i. TALLAHASSEE

      ii. ASHLAND

   (c) A baseball team has 24 players. They are staying in 6 hotel rooms, with 4 players in each room. In how many different ways can the room assignments be made? (Note. This problem and the next two may also be solved using techniques from Section 14)

   (d) In how many ways can 3 teams containing 4, 3 and 2 people be selected from a group of 9 people?

   (e) In how many ways can we place 7 identical balls in 11 distinct boxes if each box can contain no more that one ball?

   (f) In how many ways can you arrange 8 distinct keys on a key ring?

4. Section 14

   (a) How many lists of length 4 can be made from the symbols A, B, C, D, E, F, G, H, I and J if no letter can be repeated and the list must be in alphebetical order?

   (b) In how many ways can a committee of 8 men and 6 women be chosen from an organization that contains 12 men and 14 women?

   (c) Given a standard 52-card deck,

      i. How many different 7-card hands can be dealt?

      ii. How many different 7-card hands are there in which all cards are of the same suit?

      iii. How many different 7-card hands are there in which 3 cards are hearts and the others are a different suit?

   (d) A coin is tossed 12 times. In how many different ways can there be 4 heads and 8 tails? (E.g. one sequence might be HHHHTTTTTTTT and another THTHTHTHTTTT, etc.)

   (e) A coin is tossed 12 times. In how many different ways can there be 4 or *fewer* heads?

   (f) Use Pascal's triangle to expand $(x + y)^7$.

   (g) In the expansion of $(3x - 2y)^7$, find the coefficient of the x^4y^3 term.

   (h) How many 6-element subsets does a 12-element set have?

   (i) In how many ways can a group of 12 people be divided into two teams of 6 people?

   (j) Consider a regular polygon with n sides. How many diagonals are there?

5. Section 15

   (a) How many different 5-element multisets can be made from the elements $\{a, b, c, d\}$?
   (b) A bakery sells 20 different types of doughnuts. In how many different ways can one select 8 doughnuts?
   (c) How many different solutions are there to the equation $w+x+y+z = 70$ if each variable must have a nonnegative value?
   (d) How many different collections of 8 coins can be made from pennies, nickels, dimes and quarters?
   (e) You have a bag containing 50 Hershey's Kisses, 50 Snickers bars and 15 apples. A trick-or-treater takes 8 items from your bag. In how many ways can this be done?

6. Section 16

   (a) This problem concerns length-4 lists made from the letters A, B, C, D, E, with repetition allowed. How many such lists are there if the list begins with a vowel or ends with A, or consists entirely of vowels?
   (b) In a class of 200, 75 take calculus, 70 take history, 75 take sociology, 35 take calculus and sociology, 20 take history and sociology, 25 take calculus and history, and 15 take all 3.
      i. How many take at least one of the 3 subjects?
      ii. How many take exactly one of the subjects?
      iii. How many take history or calculus but not sociology?
      iv. How many take exactly 2 of the 3 subjects?
      v. How many take none of the 3 subjects?

7. Section 20

   (a) Consider the set $A = \{a, b, c, d, e\}$.
      i. How many different functions $f : A \to A$ are there?
      ii. How many different *injective* functions $f : A \to A$ are there?
      iii. How many different *non-injective* functions $f : A \to A$ are there?
      iv. How many different functions $f : A \to A$ are there satisfying $f(a) = c$?
      v. How many different functions $f : A \to A$ are there satisfying $f(a) \in \{a, c, d\}$?
      vi. How many different functions $f : A \to A$ are there satisfying $f(a) = f(b)$?
      vii. How many different functions $f : A \to A$ are there satisfying $f(a) = f(b) = c$?
      viii. How many different functions $f : A \to A$ are there with $|\{x \in A : f(x) = c\}| = 3$?
      ix. How many different functions $f : A \to A$ are there satisfying $f(a) = a$ or $f(b) = c$?
      x. How many different *surjective* functions $f : A \to A$ are there?

8. Section 21

   (a) You have a drawer full of black socks and blue socks. One dark morning the power is out and you are packing for a trip. How many socks must you pull out of the drawer to be sure that you get a matching pair? Explain how your answer illustrates the pigeonhole principle.

   (b) Show that at any party of 30 people, there are 2 that have the same number of friends at the party.

   (c) In a group of people who come from 5 different countries, how large must the group be to guarantee that 2 of them come from the same country?