

Directions: There are TWO pages. Please answer in the space provided. No calculators. Please put all phones, etc., away.

1. State the Primary Decomposition Theorem.

2. State the Cyclic Decomposition Theorem.

3. Suppose A is a 5×5 matrix with complex entries and minimum polynomial $m_A = (x - 4)(x - 1)^2$. Given that this is all you know about A , list its possible Jordan forms.

4. Let $T : \mathbb{R}^3 \rightarrow \mathbb{R}^3$ be the transformation defined as $T\alpha = A\alpha$, where $A = \begin{bmatrix} 1 & 1 & 0 \\ 2 & 2 & 1 \\ 0 & 4 & 1 \end{bmatrix}$.

The minimum polynomial of T happens to be $m_T = x^3 - 4x^2 - x + 4$.

(a) Say $\alpha_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$. Find a basis for $Z(\alpha_1; T)$.

(b) Using your answer from part (a), find the rational form of T .