Quiz 2 | Advanced Linear Algebra | October 11, 2018

MATH 610

Name: ______________________ R. Hammack Score: ________

Directions: There are TWO pages. Please answer in the space provided. No calculators. Please put all phones, etc., away.

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1. Let $A = \begin{bmatrix} 2 & 5 \\ 0 & 1 \end{bmatrix}$. Find the monic polynomial $p \in \mathbb{R}[x]$ of lowest degree for which $p(A) = O$.

   That is, find the monic generator of the ideal $I = \{f \in \mathbb{R}[x] \mid f(A) = O\}$.

2. Let $V = \{(x_1, x_2, x_3, x_4, x_5, \ldots) \mid x_i \in \mathbb{R}\}$ be the vector space (over $\mathbb{R}$) of all infinite sequences with terms in $\mathbb{R}$. Let $T : V \to V$ be defined as $T(x_1, x_2, x_3, x_4, x_5, \ldots) = (x_2, x_3, x_4, x_5, x_6, \ldots)$. That is, T shifts each term of an input sequence one position to the left, dropping the first term. Example: $T(1, 1, 2, 3, 5, 8, 13, 21, \ldots) = (1, 2, 3, 5, 8, 13, 21, \ldots)$.

   Describe the eigenvectors of T.

3. Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be the transformation defined as $T\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 & 4 \\ -1 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$.

(a) Find the eigenvalues of T.

(b) Find the eigenspaces of T.

(c) Is T diagonalizable? Explain.