Quiz 1 | Advanced Linear Algebra | September 6, 2018

MATH 610

Name: ______________________ R. Hammack Score: ________

Directions: There are TWO pages. Please answer in the space provided. No calculators. Please put all phones, etc., away.

1. State what it means for a subset S of a vector space V over $\mathbb{F}$ to be **linearly dependent**.

2. Let V be the vector space (over $\mathbb{R}$) of all functions $f : \mathbb{R} \to \mathbb{R}$.
   Let $W = \{ f \in V \mid f(-x) = f(x) \text{ for all } x \in \mathbb{R} \}$. That is, W is the set of all *even* functions in V.
   Let $X = \{ f \in V \mid f(-x) = -f(x) \text{ for all } x \in \mathbb{R} \}$. That is, X is the set of all *odd* functions in V.

   (a) Prove that W is a subspace of V. (Note that X is a also a subspace of V, but you don't need to prove it.)

   (b) Show that the set $W \cup X$ spans V.

3. Suppose V is a finite-dimensional vector space and $T : V \to V$ is a linear transformation having the property $\text{Range}(T) = \text{Null}(T)$, that is, the range of T and the null space of T are the same subspace.

   (a) Show that $\dim(V)$ is an even number.

   (b) Give an example of such a T and V.