Abstract Algebra II | MATH 602 Midterm | March 7, 2012

Name:__________________ R. Hammack Score:______

1. Determine all the ideals in the ring $\mathbb{Z}[x]/(2, x^3+1)$.

2. Construct a field with 9 elements.

3. Let z be a fixed element in the center of a ring R with 1, and let M be a (left) R-module.
   Prove: The map $\mu_z : M \to M$ given by $\mu_z(m) = zm$ is an R-module homomorphism.
   Prove: If R is commutative, then the map $\varphi : R \to \operatorname{End}_R(M)$ given by $\varphi(r) = \mu_r$ is a ring homomorphism.

4. Prove that if M is a finitely generated R-module that is generated with n elements, then every quotient of M is finitely generated by n or fewer elements.

5. Suppose V is finite dimensional vector space and $\varphi : V \to V$ is a linear transformation.
   Prove that there is an integer m for which $\varphi^m(V) \cap \ker \varphi^m = 0$.