Abstract Algebra I | MATH 601 Midterm | October 29, 2015

Name:________________ R. Hammack Score:______

1. Short Answer (8 points each)

   (a) Draw the subgroup lattice for $\mathbb{Z}/36\mathbb{Z}$.

   (b) List all Sylow subgroups of $\mathbb{Z}/36\mathbb{Z}$.

   (c) Find a representative of each conjugacy class of elements of order 4 in S_8.

   (d) State Cauchy's Theorem.

   (e) Give an example a subgroup that is normal but not characteristic.

2. Prove that $H \leq C_G(H)$ if and only if H is abelian.

3. Prove that the subgroup of S_4 generated by $(1\ 2)$ and $(1\ 3)(2\ 4)$ is isomorphic to D_8.

4. Suppose $A \trianglelefteq G$, and A is abelian. Recall that in this situation $AB \leq G$. Let $B \leq G$ be any subgroup. Prove $A \cap B \trianglelefteq AB$.

5. Suppose G is a group of odd order. Prove that for any non-identity element $x \in G$, x and x^{-1} are not conjugate in G.

6. Prove that $Z(S_n) = 1$ for all $n \geq 3$.

7. Let G be a group of order 200. Prove that G has a normal Sylow 5-subgroup.