Abstract Algebra I | MATH 601 Midterm | October 31, 2012

Name:____________________ R. Hammack Score:________

1. Short Answer (8 points each)

   (a) Draw the subgroup lattice for S_3.

   (b) List all generators of $\mathbb{Z}/54\mathbb{Z}$.

   (c) List all Sylow subgroups of $\mathbb{Z}/54\mathbb{Z}$.

   (d) Give an example of two elements of A_5 that are conjugate in S_5 but not conjugate in A_5.

   (e) Show $SL_2(\mathbb{F}_3) \ncong S_4$.

2. Prove that $Z(D_{2n}) = 1$ if n is odd.

3. Prove that if the center of G is of index n, then every conjugacy class of G has at most n elements.

4. Use Sylow's Theorem to prove Cauchy's Theorem.

5. Prove that if $H \leq G$ has index n, then there is a normal subgroup K of G with $K \leq H$ and $|G : K| \leq n!$.

6. Let G be a group and $\sigma \in \mathrm{Aut}(G)$. Suppose $\varphi_g \in \mathrm{Aut}(G)$ is conjugation by g, that is, $\varphi_g(x) = gxg^{-1}$. Prove that $\sigma\varphi_g\sigma^{-1} = \varphi_{\sigma(g)}$. Deduce that $\mathrm{Inn}(G) \trianglelefteq \mathrm{Aut}(G)$.

7. Let G be a group of order 462. Prove that G is not simple.