

Name: \_\_\_\_\_

R. Hammack

Score: \_\_\_\_\_

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1. Short Answer (8 points each)

(a) Draw the subgroup lattice for  $Q_8$ .

(b) Find the order of  $\overline{30}$  in  $\mathbb{Z}/54\mathbb{Z}$ .

(c) State the class equation.

(d) Write down the elements of a Sylow 2-subgroup of  $A_4$ .

(e) Give an example of a non-abelian group that is simple.

2. Suppose  $n \geq 3$ . Show that the set  $A = \{x \in D_{2n} \mid x^2 = 1\}$  is not a subgroup of  $D_{2n}$ .

3. Prove the multiplicative group  $\mathbb{Q}^+$  of positive rational numbers is generated by the set  $A = \left\{ \frac{1}{p} \mid p \text{ is prime} \right\}$ .

4. Prove that if  $G/Z(G)$  is cyclic, then  $G$  is abelian.

5. Prove that if  $|G : H| = 2$ , then  $H \trianglelefteq G$ .

6. Prove that characteristic subgroups are normal.

7. Prove that a group of order 56 has a normal Sylow  $p$ -group for some prime  $p$  dividing its order.