Name: ⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯   Score: ⎯⎯⎯⎯⎯

Directions: This is a closed-book, closed-notes test. You *may not* use calculators, computers, etc.

1. (12 points) Is the sequence $6, 6, 5, 4, 3, 2, 2$ graphical?

2. (12 points) Prove that if G is a graph with minimum degree $\delta(G) \geq 2$ then G contains a cycle.

3. (12 points) Show that for every two vertices u, v in a connected graph G, there is a u-v walk containing all vertices of G.

4. (12 points) Construct a tree with Prüfer code (1,8,1,5,2,5).

5. (12 points) Let G be a connected graph of order 3 or more. Prove that if $e = uv$ is a bridge of G then at least one of u or v is a cut vertex of G.

6. (12 points) Find necessary and sufficient conditions on G and H that guarantee that the Cartesian product $G \square H$ is Eulerian. Justify your answer.

7. (14 points) Let G be a graph of order $n \geq 3$ having the property that for each vertex $x \in V(G)$, there is a Hamiltonian path with initial vertex x. Show that G is 2-connected but not necessarily Hamiltonian.

8. (14 points) Prove or disprove: If every vertex of a tournament belongs to a cycle, then the tournament is strong.