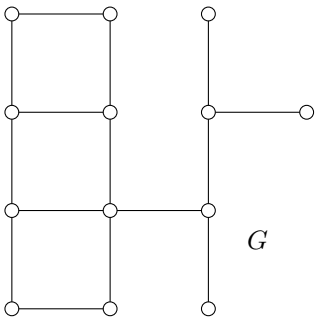
MATH 591: Graph Theory | Test | October 15, 2010

Name: _____ Score: _____

Directions: This is a closed-book, closed-notes test. Please answer in the space provided.
You *may not* use calculators, computers, etc.

1. (15 points) A graph G is drawn below. Label each vertex with its eccentricity.
State the radius and diameter of G. Indicate the center of G.



2. (15 points) Suppose $k \geq 2$. Prove that a k-regular bipartite graph has no cut-edge.

3. (15 points) Let $k \geq 2$ be a fixed integer.
Suppose a tree T has p vertices of degree k, and all the other vertices of T have degree 1. Find $n(T)$.

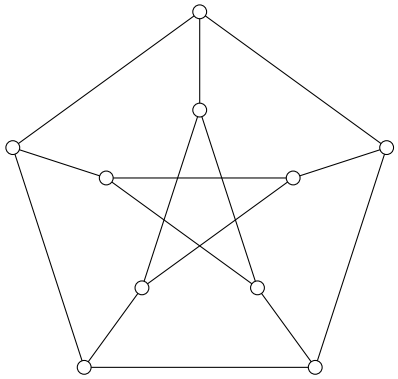
4. (15 points) State the following theorems carefully and precisely.

   (a) Berge's Theorem

   (b) Hall's Theorem

   (c) The König-Egervary Theorem

5. (20 points) Find the listed invariants for the Petersen graph.



(a) $\alpha =$

(b) $\gamma =$

(c) $\alpha' =$

(d) $\chi =$

(e) $\omega =$

6. (10 points) Prove that $\gamma \leq \alpha$ for any graph.

7. (10 points) Prove that $\chi \cdot \alpha \geq n$ for any graph.