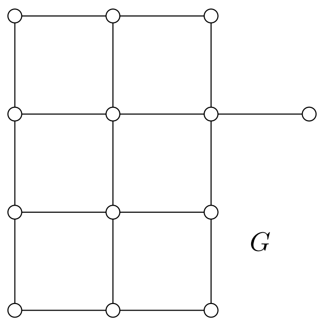
MATH 591: Graph Theory | Test | October 19, 2009

Name: ____________________ Score: ________

**Directions:** This is a closed-book, closed notes test. Please answer in the space provided. You *may not* use calculators, computers, etc.

1. (16 points) A graph $G$ is drawn below. Label each vertex with its eccentricity.
State the radius and diameter of $G$. Indicate the center of $G$.



$G$

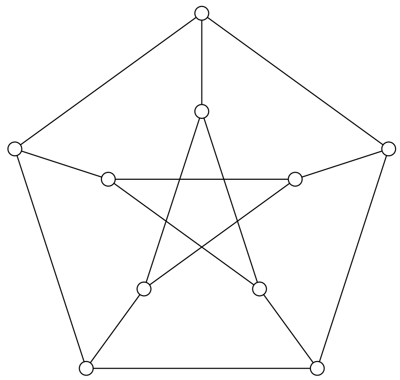
2. (6 points) Decide if the sequence $s: \; 4 \; 4 \; 4 \; 4 \; 3 \; 2$ is graphical. Show your work and/or explain your reasoning.

3. (16 points) Suppose $G$ is a graph of order $n$, and $\deg(v) \geq \dfrac{n-1}{2}$ for every $v \in V(G)$. Prove that $G$ is connected.

4. (10 points) Let $G$ be a connected graph on at least three vertices, and let $e = uv$ be a bridge of $G$. Show that either $u$ or $v$ is a cut vertex of $G$.

5. (12 points) What does it mean for a graph to be reconstructible? Give an example (with explanation) of a graph of small order that is not reconstructible.

6. (12 points) Consider the Petersen Graph, sketched below.



Supply the following numeric information. (For *this* problem you do not have to justify your answers.)

(a) The connectivity is $\kappa(G) =$

(b) The edge-connectivity $\kappa_1(G) =$

(c) The toughness is $t(G) =$

7. (6 points) Suppose a forest has 1000 vertices and 800 edges. How many components does it have?

8. (12 points) Suppose $G$ is a planar graph with 16 vertices, each of degree 4. It is embedded in the plane so that every region is either a triangle or a quadrangle. How many triangles and how many quadrangles does this embedding have? Explain.

9. (10 points) Establish the planarity or non-planarity of this graph. If it is planar, provide a rectilinear plane drawing.