VCU

MATH 525
COMBINATORICS

R. Hammack

TEST 2

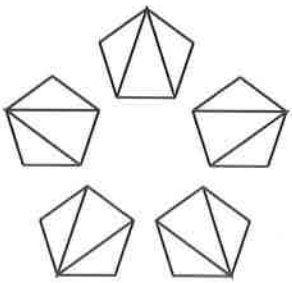
April 7, 2016

Name: _____

Directions. Answer the questions in the space provided. Justify each step to the extent reasonable.

This is a closed-book, closed-notes test.

There are 5 numbered questions; each is worth 20 points.



1. Solve the recurrence relation $h_n = 4h_{n-2}$ with initial values $h_0 = 0$ and $h_1 = 1$.

2. Solve the recurrence relation $h_n = 2h_{n-1} + n$ with initial value $h_0 = 1$.

3. Use generating functions to find how many ways there are to put n identical balls into four boxes, in such a way that the first box has no more than 3 balls, the second has a multiple of 4 balls, the third has at least 5 balls, and there is no restriction on the number of balls in the fourth box.

4. Let h_n be the number of ways to color the squares of a $1 \times n$ chessboard red, white, blue & green so there are an even number of red squares and an odd number of white ones. Find the exponential generating function for the sequence $h_0, h_1, h_2, \ldots$. Use it to find a simple formula for h_n.

5. Find the (ordinary) generating function for the infinite sequence $h_0, h_1, h_2, \ldots$ defined by $h_n = \binom{n}{2}$.