Abstract Algebra — Test #1 — October 7, 2010

MATH 501 — R. Hammack — Name ______________________

Directions: Answer each question in the space provided. Use of any electronic device (calculators, i-pods, etc.) is not allowed during this test.

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1. (30 points) Short Answer. You do not need to show your work for problems on this page.

   (a) List the generators of $\mathbb{Z}_{12}$.

   (b) Write a multiplication table for $U(12)$.

   (c) $|A_n| =$

   (d) A group has 45 elements. What are the possible orders of its subgroups?

   (e) Suppose a is a generator of a cyclic group G. Give a generator for the subgroup $\langle a^m \rangle \cap \langle a^n \rangle$.

   (f) Give an example of a nontrivial abelian subgroup of a non-abelian group.

   (g) Write $\mu = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 5 & 1 & 6 & 7 & 2 & 4 & 3 \end{pmatrix} \in S_7$ as a product of disjoint cycles.

   (h) Is the permutation μ from part (g) even or odd?

   (i) Find the order of the permutation μ from part (g).

   (j) Write the following as a product of disjoint cycles: $(215)(3142)^{-1}$

2. (10 points) List the left cosets of the subgroup $H = \{(1), (13)\}$ of S_3.

3. (10 points) Find all possible orders of elements in S_7.

4. (10 points) Prove that if a group G has no proper nontrivial subgroups, then G is cyclic.

5. (10 points) Suppose a group G has the property that $a^2 = e$ for every $a \in G$. Prove that G is abelian.

6. (10 points) Let g be an element of a group G, and define a map $\lambda_g : G \to G$ as $\lambda_g(x) = gx$. Show that λ_g is a permutation of the set G.

7. (10 points) Let G be an **abelian** group. Show that the set of elements of finite order in G form a subgroup.

8. (10 points) Suppose H is a subgroup of a group G, and $[G : H] = 2$.
   Suppose also that a and b are in G, but not in H. Show that $ab \in H$.