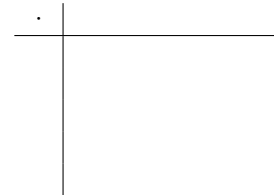


**Directions:** Answer each question in the space provided. Use of any electronic device (calculators, i-pods, etc.) is not allowed during this test.

1. (30 points) Short Answer. You do not need to show your work for problems on this page.

(a) List the generators of  $\mathbb{Z}_{12}$ .

(b) Write a multiplication table for  $U(12)$ .



(c)  $|A_n| =$

(d) A group has 45 elements. What are the possible orders of its subgroups?

(e) Suppose  $a$  is a generator of a cyclic group  $G$ . Give a generator for the subgroup  $\langle a^m \rangle \cap \langle a^n \rangle$ .

(f) Give an example of a nontrivial abelian subgroup of a non-abelian group.

(g) Write  $\mu = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 5 & 1 & 6 & 7 & 2 & 4 & 3 \end{pmatrix} \in S_7$  as a product of disjoint cycles.

(h) Is the permutation  $\mu$  from part (g) even or odd?

(i) Find the order of the permutation  $\mu$  from part (g).

(j) Write the following as a product of disjoint cycles:  $(215)(3142)^{-1}$

2. (10 points) List the left cosets of the subgroup  $H = \{(1), (13)\}$  of  $S_3$ .

3. (10 points) Find all possible orders of elements in  $S_7$ .

4. (10 points) Prove that if a group  $G$  has no proper nontrivial subgroups, then  $G$  is cyclic.

5. (10 points) Suppose a group  $G$  has the property that  $a^2 = e$  for every  $a \in G$ . Prove that  $G$  is abelian.

6. (10 points) Let  $g$  be an element of a group  $G$ , and define a map  $\lambda_g : G \rightarrow G$  as  $\lambda_g(x) = gx$ . Show that  $\lambda_g$  is a permutation of the set  $G$ .

7. (10 points) Let  $G$  be an **abelian** group. Show that the set of elements of finite order in  $G$  form a subgroup.

8. (10 points) Suppose  $H$  is a subgroup of a group  $G$ , and  $[G : H] = 2$ .  
Suppose also that  $a$  and  $b$  are in  $G$ , but not in  $H$ . Show that  $ab \in H$ .