Linear Algebra	Test #3	April 25, 2007
Name: ________________	R. Hammack	Score: ______

Directions: This is a take-home test. It is due at the beginning of class on Monday, April 30. Please answer all questions in the space provided. Consider working the problems on scratch paper, then rewriting them neatly on the test. Additional copies of this test can be downloaded from my web page if needed.

For this test, you may discuss the problems among yourselves and share ideas, but the work you turn in must be your own. At the end of your solution to each problem, please list who (if anyone) you talked to about that problem, plus any additional information you want me to know.

- Where appropriate, indicate your final answer clearly by putting it in a box.
- Please indicate any row operation that you use (e.g. $R_2 + 3R_1 \to R_2$, etc.).
- You may consult your text and notes, but **no** other source.
- In order to get full credit, you must show or explain all of your work.

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1. (7 points) Suppose $T : V \to V$ is a linear transformation. Show that the set $W = \{\mathbf{x} \in V \ : \ T(\mathbf{x}) = -\mathbf{x}\}$ is a subspace of V.

   (a) First we show W is closed under addition.
   Suppose $\mathbf{u}$ and $\mathbf{v}$ are in W.
   By definition of W, this means $T(\mathbf{u}) = -\mathbf{u}$ and $T(\mathbf{v}) = -\mathbf{v}$.
   Using this together with the fact that T is linear gives $T(\mathbf{u}+\mathbf{v}) = T(\mathbf{u}) + T(\mathbf{v}) = -\mathbf{u} - \mathbf{v} = -(\mathbf{u}+\mathbf{v})$.
   Therefore we have $T(\mathbf{u}+\mathbf{v}) = -(\mathbf{u}+\mathbf{v})$, and this means $\mathbf{u}+\mathbf{v} \in W$.
   Therefore W is closed under addition.

   (b) Next we show W is closed under scalar multiplication.
   Suppose $\mathbf{u}$ is in W and $c \in \mathbb{R}$.
   By definition of W, the fact that $\mathbf{u} \in W$ means $T(\mathbf{u}) = -\mathbf{u}$.
   Using this together with the fact that T is linear gives $T(c\mathbf{u}) = cT(\mathbf{u}) = c(-\mathbf{u}) = -(c\mathbf{u})$.
   Therefore we have $T(c\mathbf{u}) = -(c\mathbf{u})$, and this means $c\mathbf{u} \in W$.
   Therefore W is closed under scalar multiplication.

   From (a) and (b) above, it follows that W is a subspace of V.

2. (7 points) Find the dimension of the subspace of $\mathbb{R}^4$ spanned by the vectors $(7,6,5,7)$, $(1,1,2,1)$, $(3,2,1,4)$, $(3,3,2,2)$, and $(4,3,-1,4)$.

   This subspace is the row space of $\begin{bmatrix} 7 & 6 & 5 & 7 \\ 1 & 1 & 2 & 1 \\ 3 & 2 & 1 & 4 \\ 3 & 3 & 2 & 2 \\ 4 & 3 & -1 & 4 \end{bmatrix}$. We can find the dimension by row reducing.

$$\begin{bmatrix} 7 & 6 & 5 & 7 \\ 1 & 1 & 2 & 1 \\ 3 & 2 & 1 & 4 \\ 3 & 3 & 2 & 2 \\ 4 & 3 & -1 & 4 \end{bmatrix} \begin{matrix} R_1 \leftrightarrow R_2 \end{matrix} \begin{bmatrix} 1 & 1 & 2 & 1 \\ 7 & 6 & 5 & 7 \\ 3 & 2 & 1 & 4 \\ 3 & 3 & 2 & 2 \\ 4 & 3 & -1 & 4 \end{bmatrix} \begin{matrix} R_2 - 7R_1 \to R_2 \\ R_3 - 3R_1 \to R_3 \\ R_4 - 3R_1 \to R_4 \\ R_5 - 4R_1 \to R_5 \end{matrix} \begin{bmatrix} 1 & 1 & 2 & 1 \\ 0 & -1 & -9 & 0 \\ 0 & -1 & -5 & 1 \\ 0 & 0 & -4 & -1 \\ 0 & -1 & -9 & 0 \end{bmatrix} \begin{matrix} R_3 - R_2 \to R_3 \\ R_5 - R_2 \to R_5 \end{matrix}$$

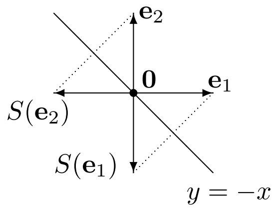
$$\begin{bmatrix} 1 & 1 & 2 & 1 \\ 0 & -1 & -9 & 0 \\ 0 & 0 & 4 & 1 \\ 0 & 0 & -4 & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{matrix} R_1 + R_2 \to R_1 \\ R_4 + R_3 \to R_4 \end{matrix} \begin{bmatrix} 1 & 0 & -7 & 1 \\ 0 & -1 & -9 & 0 \\ 0 & 0 & 4 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{matrix} -R_2 \to R_2 \\ \\ \frac{1}{4}R_3 \to R_3 \end{matrix} \begin{bmatrix} 1 & 0 & -7 & 1 \\ 0 & 1 & 9 & 0 \\ 0 & 0 & 1 & \frac{1}{4} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{matrix} R_1 + 7R_3 \to R_1 \\ R_2 - 9R_3 \to R_2 \end{matrix} \begin{bmatrix} 1 & 0 & 0 & 11/4 \\ 0 & 1 & 9 & -9/4 \\ 0 & 0 & 1 & 1/4 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

   At this point you can see that the reduced matrix has three nonzero rows, and these form a basis for the row space, by Theorem 14.14. Therefore **The subspace spanned by the vectors is three-dimensional.**

3. For the questions on this page $S : \mathbb{R}^2 \to \mathbb{R}^2$ is the linear transformation for which $S(\mathbf{x})$ is the reflection of $\mathbf{x}$ across the line $y = -x$, and $T : \mathbb{R}^2 \to \mathbb{R}^2$ is rotation by 30° clockwise.

(a) (5 points) Find the standard matrix for S.



Notice from the picture we can see that $S(\mathbf{e}_1) = -\mathbf{e}_2$ and $S(\mathbf{e}_2) = -\mathbf{e}_1$.

Therefore $A = [S(\mathbf{e}_1)\ S(\mathbf{e}_2)] = [-\mathbf{e}_2\ \ -\mathbf{e}_1] = \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$

(b) (5 points) Find the standard matrix for T.

Using standard ideas from trigonometry, we get

$$B = [T(\mathbf{e}_1)\ T(\mathbf{e}_2)] = \begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$$

(c) (5 points) Find the standard matrix for $S \circ T$.

$$AB = \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & -\frac{1}{2} \end{bmatrix}$$

(d) (5 points) Find the standard matrix for $(S \circ T)^{-1}$.

$$\begin{bmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & -\frac{1}{2} \end{bmatrix}^{-1} = \frac{1}{-\frac{1}{2}\frac{1}{2} - \frac{\sqrt{3}}{2}\frac{\sqrt{3}}{2}} \begin{bmatrix} -\frac{1}{2} & \frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & -\frac{1}{2} \end{bmatrix}$$

4. The questions on this page concern the set $B = \{(1,2,2,1), (-1,1,2,4), (1,1,0,1), (3,1,2,3)\}$.

(a) (7 points) Verify that B is a basis for $\mathbb{R}^4$.

To check for independence, we need to investigate the solutions to the equation $w(1,2,2,1) + x(-1,1,2,4) + y(1,1,0,1) + z(3,1,2,3) = (0,0,0,0)$.

This gives rise to the system $\begin{cases} w - x + y + 3z &= 0 \\ 2w + x + y + z &= 0 \\ 2w + 2x + 0y + 2z &= 0 \\ w + 4x + y + 3z &= 0 \end{cases}$

$$\begin{bmatrix} 1 & -1 & 1 & 3 & 0 \\ 2 & 1 & 1 & 1 & 0 \\ 2 & 2 & 0 & 2 & 0 \\ 1 & 4 & 1 & 3 & 0 \end{bmatrix} \begin{matrix} R_2 + 2R_1 \to R_2 \\ R_3 + 2R_1 \to R_3 \\ R_4 + R_1 \to R_4 \end{matrix} \begin{bmatrix} 1 & -1 & 1 & 3 & 0 \\ 0 & 3 & -1 & -5 & 0 \\ 0 & 4 & -2 & -4 & 0 \\ 0 & 5 & 0 & 0 & 0 \end{bmatrix} \begin{matrix} \frac{1}{2}R_3 \to R_3 \\ \\ \frac{1}{5}R_4 \to R_4 \end{matrix} \begin{bmatrix} 1 & -1 & 1 & 3 & 0 \\ 0 & 3 & -1 & -5 & 0 \\ 0 & 2 & -1 & -2 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{matrix} R_1 + R_4 \to R_1 \\ R_2 - 3R_4 \to R_2 \\ R_3 - 2R_4 \to R_3 \end{matrix} \begin{bmatrix} 1 & 0 & 1 & 3 & 0 \\ 0 & 0 & -1 & -5 & 0 \\ 0 & 0 & -1 & -2 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix} R_2 \leftrightarrow R_4 \begin{bmatrix} 1 & 0 & 1 & 3 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & -2 & 0 \\ 0 & 0 & -1 & -5 & 0 \end{bmatrix} R_4 - R_3 \to R_3$$

$$\begin{bmatrix} 1 & 0 & 1 & 3 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & -2 & 0 \\ 0 & 0 & 0 & -3 & 0 \end{bmatrix} \begin{matrix} R_1 + R_3 \to R_1 \\ -R_3 \to R_3 \\ -\frac{1}{3}R_4 \to R_4 \end{matrix} \begin{bmatrix} 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} \begin{matrix} R_3 - \frac{2}{3}R_4 + R_3 \to R_3 \\ R_1 - R_4 + R_3 \to R_1 \end{matrix} \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

Therefore there are no nontrivial solutions, so the set B is linearly independent.
Since B is a linearly independent set of 4 vectors, and $\mathbb{R}^4$ has dimension 4, it follows by Theorem 4.12 that B is a basis for $\mathbb{R}^4$.

(b) (7 points) Given that $\mathbf{x} = (1,0,0,1)$, find $[\mathbf{x}]_B$.

$$[\mathbf{x}]_B = \begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} \quad \text{where} \quad w(1,2,2,1) + x(-1,1,2,4) + y(1,1,0,1) + z(3,1,2,3) = (1,0,0,1).$$

This gives rise to the system $\begin{cases} w - x + y + 3z &= 1 \\ 2w + x + y + z &= 0 \\ 2w + 2x + 0y + 2z &= 0 \\ w + 4x + y + 3z &= 1 \end{cases}$

$$\begin{bmatrix} 1 & -1 & 1 & 3 & 1 \\ 2 & 1 & 1 & 1 & 0 \\ 2 & 2 & 0 & 2 & 0 \\ 1 & 4 & 1 & 3 & 1 \end{bmatrix} \begin{matrix} R_2 + 2R_1 \to R_2 \\ R_3 + 2R_1 \to R_3 \\ R_4 + R_1 \to R_4 \end{matrix} \begin{bmatrix} 1 & -1 & 1 & 3 & 1 \\ 0 & 3 & -1 & -5 & -2 \\ 0 & 4 & -2 & -4 & -2 \\ 0 & 5 & 0 & 0 & 0 \end{bmatrix} \begin{matrix} \frac{1}{2}R_3 \to R_3 \\ \\ \frac{1}{5}R_4 \to R_4 \end{matrix} \begin{bmatrix} 1 & -1 & 1 & 3 & 1 \\ 0 & 3 & -1 & -5 & -2 \\ 0 & 2 & -1 & -2 & -1 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{matrix} R_1 + R_4 \to R_1 \\ R_2 - 3R_4 \to R_2 \\ R_3 - 2R_4 \to R_3 \end{matrix} \begin{bmatrix} 1 & 0 & 1 & 3 & 1 \\ 0 & 0 & -1 & -5 & -2 \\ 0 & 0 & -1 & -2 & -1 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix} R_2 \leftrightarrow R_4 \begin{bmatrix} 1 & 0 & 1 & 3 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & -2 & -1 \\ 0 & 0 & -1 & -5 & -2 \end{bmatrix} R_4 - R_3 \to R_3$$

$$\begin{bmatrix} 1 & 0 & 1 & 3 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & -2 & -1 \\ 0 & 0 & 0 & -3 & -1 \end{bmatrix} \begin{matrix} R_1 + R_3 \to R_1 \\ -R_3 \to R_3 \\ -\frac{1}{3}R_4 \to R_4 \end{matrix} \begin{bmatrix} 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 2 & 1 \\ 0 & 0 & 0 & 1 & 1/3 \end{bmatrix} \begin{matrix} R_3 - \frac{2}{3}R_4 + R_3 \to R_3 \\ R_1 - R_4 + R_3 \to R_1 \end{matrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & -1/3 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1/3 \\ 0 & 0 & 0 & 1 & 1/3 \end{bmatrix}$$

Therefore $\boxed{[\mathbf{x}]_B = \begin{bmatrix} -1/3 \\ 0 \\ 1/3 \\ 1/3 \end{bmatrix}}$

5. Questions on this page involve the transformation $T : \mathbb{R}^3 \to \mathbb{R}^4$ defined as $T\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} x+y \\ x-z \\ y+z \\ z-x \end{bmatrix}$.

(a) (5 points) Find the standard matrix for T.

$$\boxed{A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & -1 \\ 0 & 1 & 1 \\ -1 & 0 & 1 \end{bmatrix}} \text{ because } \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & -1 \\ 0 & 1 & 1 \\ -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} x+y \\ x-z \\ y+z \\ z-x \end{bmatrix}.$$

(b) (7 points) Find the kernel of T.

The kernel of T is the nullspace of A, and that is the set of solutions to $\begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & -1 \\ 0 & 1 & 1 \\ -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$.

Solving in the usual way, we get

$$\begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 0 & -1 & 0 \\ 0 & 1 & 1 & 0 \\ -1 & 0 & 1 & 0 \end{bmatrix} \begin{matrix} R_2 - R_1 \to R_2 \\ R_3 + R_1 \to R_3 \end{matrix} \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & -1 & -1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix} \begin{matrix} R_3 + R_2 \to R_3 \\ R_4 + R_2 \to R_4 \\ R_1 + R_2 \to R_1 \\ -R_2 \to R_2 \end{matrix} \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{matrix} x = z \\ y = -z \\ z = free \end{matrix}$$

Thus $\boxed{\text{Ker}(T) = \left\{ \begin{bmatrix} t \\ -t \\ t \end{bmatrix} : t \in \mathbb{R} \right\} = \text{span}\left(\begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}\right)}$

(c) (5 points) Find the rank of T.

Part (b) above tells us that the kernel is one-dimensional, so $\text{nullity}(T) = 1$
Theorem 6.5 tells us $\text{rank}(T) + \text{nullity}(T) = \dim(\mathbb{R}^3)$.
Therefore $\text{rank}(T) + 1 = 3$, so $\boxed{\text{rank}(T) = 2.}$

(d) (7 points) Find the preimage of $\begin{bmatrix} 1 \\ 1 \\ 0 \\ -1 \end{bmatrix}$.

This is the set of all vectors $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$ for which $\begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & -1 \\ 0 & 1 & 1 \\ -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 0 \\ -1 \end{bmatrix}$.

Solving in the usual way, we get

$$\begin{bmatrix} 1 & 1 & 0 & 1 \\ 1 & 0 & -1 & 1 \\ 0 & 1 & 1 & 0 \\ -1 & 0 & 1 & -1 \end{bmatrix} \begin{matrix} R_2 - R_1 \to R_2 \\ R_3 + R_1 \to R_3 \end{matrix} \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & -1 & -1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix} \begin{matrix} R_3 + R_2 \to R_3 \\ R_4 + R_2 \to R_4 \\ R_1 + R_2 \to R_1 \\ -R_2 \to R_2 \end{matrix} \begin{bmatrix} 1 & 0 & -1 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{matrix} x = 1 + z \\ y = -z \\ z = free \end{matrix}$$

Thus the preimage is the set of vectors $\boxed{\left\{ \begin{bmatrix} 1+t \\ -t \\ t \end{bmatrix} : t \in \mathbb{R} \right\}}$

6. (7 points) Suppose $T : P_2 \to P_2$ is a linear transformation satisfying $T(3 - x + 4x^2) = 1 + x - x^2$ and $T(2 - 3x + 2x^2) = 7 + 3x + 2x^2$. Find $T(7x + 2x^2)$.

The idea is to find real numbers a and b for which $7x + 2x^2 = a(3 - x + 4x^2) + b(2 - 3x + 2x^2)$, for then the unknown $T(7x + 2x^2)$ could be expressed in terms of the known $T(3 - x + 4x^2)$ and $T(2 - 3x + 2x^2)$.

Now $7x + 2x^2 = a(3 - x + 4x^2) + b(2 - 3x + 2x^2)$ simplifies to $7x + 2x^2 = (3a + 2b) + (-a - 3b)x + (4a + 2b)x^2$.

This gives rise to the system $\begin{cases} 3a + 2b &= 0 \\ -a - 3b &= 7 \\ 4a + 2b &= 2 \end{cases}$

Solving in the usual way gives $\begin{bmatrix} 3 & 2 & 0 \\ -1 & -3 & 7 \\ 4 & 2 & 2 \end{bmatrix} \xrightarrow{\substack{R_1 \leftrightarrow R_2 \\ \frac{1}{2}R_3 \to R_3}} \begin{bmatrix} -1 & -3 & 7 \\ 3 & 2 & 0 \\ 2 & 1 & 1 \end{bmatrix} \xrightarrow{\substack{R_2 + 3R_1 \to R_2 \\ R_3 + 2R_1 \to R_3}} \begin{bmatrix} -1 & -3 & 7 \\ 0 & -7 & 21 \\ 0 & -5 & 15 \end{bmatrix}$

$\xrightarrow{\substack{-\frac{1}{7}R_2 \to R_2 \\ -\frac{1}{5}R_3 \to R_3}} \begin{bmatrix} -1 & -3 & 7 \\ 0 & 1 & -3 \\ 0 & 1 & -3 \end{bmatrix} \xrightarrow{\substack{R_1 + 3R_2 \to R_1 \\ R_3 - R_2 \to R_3}} \begin{bmatrix} -1 & 0 & -2 \\ 0 & 1 & -3 \\ 0 & 0 & 0 \end{bmatrix} \xrightarrow{\frac{1}{2}R_1 \to R_1} \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & -3 \\ 0 & 0 & 0 \end{bmatrix}$ $\quad a = 2$, $b = -3$

Thus $T(7x + 2x^2) = T(\ 2(3 - x + 4x^2) - 3(2 - 3x + 2x^2)\ ) = 2T(3 - x + 4x^2) - 3T(2 - 3x + 2x^2)$
$= 2(1 + x - x^2) - 3(7 + 3x + 2x^2) = \boxed{-19 - 7x - 8x^2.}$

7. The set $B = \{1, x, xe^{3x}, e^{3x}\}$ is a basis for a subspace of W of $C(-\infty, \infty)$.
Let $T : W \to W$ be the linear transformation $T(f) = D_x[f]$ (where $D_x[f]$ is the derivative of f).

(a) (7 points) Find kernel of T.

Since the derivative of a function is zero if and only if the function is constant,
the kernel of T is the set of all constant functions. $\boxed{\text{Ker}(T)=\{a + 0x + 0xe^{3x} + 0e^{3x} \ : \ a \in \mathbb{R}\}}$

(b) (7 points) Find the rank of T.

Part (a) above tells us that the kernel is one-dimensional, so nullity$(T) = 1$
Theorem 6.5 tells us rank(T) + nullity(T) = dim(W).
Also, W is four-dimensional because it has a basis consisting of four elements.
Therefore rank$(T) + 1 = 4$, so $\boxed{\text{rank}(T) = 3.}$

(c) (7 points) Find the matrix for T relative to the basis B.

$$\begin{aligned} A &= [\ [T(1)]_B\ [T(x)]_B\ [T(xe^{3x})]_B\ [T(e^{3x})]_B\ ] \\ &= [\ [D_x(1)]_B\ [D_x(x)]_B\ [D_x(xe^{3x})]_B\ [D_x(e^{3x})]_B\ ] \\ &= [\ [0]_B\ \ [1]_B\ \ [e^{3x} + 3xe^{3x})]_B\ \ [3e^{3x}]_B\ ] \\ &= \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 1 & 3 \end{bmatrix} \end{aligned}$$