Linear Algebra	Test #3	April 25, 2007
Name: ____________________	R. Hammack	Score: ______

Directions: This is a take-home test. It is due at the beginning of class on Monday, April 30. Please answer all questions in the space provided. Consider working the problems on scratch paper, then rewriting them neatly on the test. Additional copies of this test can be downloaded from my web page if needed.

For this test, you may discuss the problems among yourselves and share ideas, but the work you turn in must be your own. At the end of your solution to each problem, please list who (if anyone) you talked to about that problem, plus any additional information you want me to know.

- Where appropriate, indicate your final answer clearly by putting it in a $\boxed{\text{box}}$.
- Please indicate any row operation that you use (e.g. $R_2 + 3R_1 \rightarrow R_2$, etc.).
- You may consult your text and notes, but **no** other source.
- In order to get full credit, you must show or explain all of your work.

1. (7 points) Suppose $T : V \rightarrow V$ is a linear transformation. Show that the set $W = \{\mathbf{x} \in V \ : \ T(\mathbf{x}) = -\mathbf{x}\}$ is a subspace of V.

2. (7 points) Find the dimension of the subspace of $\mathbb{R}^4$ spanned by the vectors $(7, 6, 5, 7)$, $(1, 1, 2, 1)$, $(3, 2, 1, 4)$, $(3, 3, 2, 2)$, and $(4, 3, -1, 4)$.

3. For the questions on this page $S : \mathbb{R}^2 \to \mathbb{R}^2$ is the linear transformation for which $S(\mathbf{x})$ is the reflection of $\mathbf{x}$ across the line $y = -x$, and $T : \mathbb{R}^2 \to \mathbb{R}^2$ is rotation by $30°$ clockwise.

   (a) (5 points) Find the standard matrix for S.

   (b) (5 points) Find the standard matrix for T.

   (c) (5 points) Find the standard matrix for $S \circ T$.

   (d) (5 points) Find the standard matrix for $(S \circ T)^{-1}$.

4. The questions on this page concern the set $B = \{(1,2,2,1), (-1,1,2,4), (1,1,0,1), (3,1,2,3)\}$.

(a) (7 points) Verify that B is a basis for $\mathbb{R}^4$.

(b) (7 points) Given that $\mathbf{x} = (1,0,0,1)$, find $[\mathbf{x}]_B$.

5. Questions on this page involve the transformation $T : \mathbb{R}^3 \to \mathbb{R}^4$ defined as $T\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} x+y \\ x-z \\ y+z \\ z-x \end{bmatrix}$.

(a) (5 points) Find the standard matrix for T.

(b) (7 points) Find the kernel of T.

(c) (5 points) Find the rank of T.

(d) (7 points) Find the preimage of $\begin{bmatrix} 1 \\ 1 \\ 0 \\ -1 \end{bmatrix}$.

6. (7 points) Suppose $T : P_2 \to P_2$ is a linear transformation satisfying $T(3 - x + 4x^2) = 1 + x - x^2$ and $T(2 - 3x + 2x^2) = 7 + 3x + 2x^2$. Find $T(7x + 2x^2)$.

7. The set $B = \{1, x, xe^{3x}, e^{3x}\}$ is a basis for a subspace of W of $C(-\infty, \infty)$.
Let $T : W \to W$ be the linear transformation $T(f) = D_x[f]$ (where $D_x[f]$ is the derivative of f).

(a) (7 points) Find kernel of T.

(b) (7 points) Find the rank of T.

(c) (7 points) Find the matrix for T relative to the basis B.