Linear Algebra | Test #3 (Chapter 3) | October 9, 2006

Name: ______________________ R. Hammack Score: ______

Directions: This is a take-home test. It is due at the beginning of class on Wednesday, October 11. Please answer all questions in the space provided. Consider working the problems on scratch paper, then rewriting them neatly on the test. Additional copies of this test can be downloaded from my web page if needed.

For this test, you may discuss the problems among yourselves and share ideas, but the work you turn in must be your own (not copied). At the end of your solution to each problem, please list who (if anyone) you talked to about that problem, plus any additional information you want me to know (i.e. that you *gave* more help than you *received*, or vice versa, etc.).

- In order to get full credit, you must show all of your work.
- When appropriate, indicate your final answer clearly by putting it in a $\boxed{\text{box}}$.
- Constants that are not integers should be expressed as fractions.
- You may consult your text and notes, but **no** other source.
- Each problem is worth 10 points.

1. Use any method discussed in class to find $\begin{vmatrix} 2 & -1 & -2 & 5 \\ 3 & 3 & 6 & 1 \\ 0 & 1 & 3 & 0 \\ -2 & 4 & 8 & -2 \end{vmatrix}$.

2. Use any method discussed in class to find the determinant of $A = \begin{bmatrix} 1 & 0 & 1 & 4 & -4 \\ 2 & 0 & 3 & 9 & 1 \\ 7 & 3 & 5 & 9 & 0 \\ -1 & 0 & 2 & 8 & 2 \\ -1 & 0 & 0 & 2 & 1 \end{bmatrix}$.

3. Suppose A, B and C are 5×5 matrices for which $\det(A) = \frac{1}{2}$, $\det(B) = -5$ and $\det(C) = 7$.
Compute the determinant of the matrix $C(3A)^{-1}B^{-1}$.

4. A square matrix A is called an *orthogonal matrix* if $AA^T = I$.
If A is orthogonal, what are the possible values for $\det(A)$?

5. Suppose A is a square matrix, and $A^2 = A$.
What are the possible values for $\det(A)$? Explain.

6. Suppose A is an $n \times n$ matrix, and $\det(A^{-1}) = \frac{1}{3}$, and $\det(2A) = 384$.

   Find the dimension n of A. Explain your reasoning.

7. Find all values of a that make $\begin{bmatrix} a & a & 0 \\ a^2 & 2 & a \\ 0 & a & a \end{bmatrix}$ singular.

8. Consider the matrix equation $\begin{bmatrix} 12 & -15 & 0 & 5 & -8 \\ -4 & 0 & 1 & 0 & 10 \\ 20 & 3 & 5 & -1 & 0 \\ 14 & -12 & 9 & 4 & 12 \\ 0 & 9 & 10 & -3 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$.

   Explain how you know this has more than one solution without making any explicit calculations.

The problems on this page concern the matrix $A = \begin{bmatrix} 37 & 105 \\ -14 & -40 \end{bmatrix}$.

9. Find the eigenvalues of A.

10. For each eigenvalue from Question 9, find the corresponding eigenvectors.