Linear Algebra | Test #2 | March 26, 2007

Name: _____ Score: _____

**Directions:** Please answer the questions in the space provided. To get full credit you must show all of your work. Use of calculators and other computing or communication devices is **not** allowed on this test.

1. (5 points) Consider the vectors $\mathbf{u} = (8, -2, 0, 1)$, $\mathbf{v} = (3, -2, 1, 1)$ and $\mathbf{w} = (1, 4, 4, 1)$. in $\mathbb{R}^4$. Find $\mathbf{x}$, given that $\mathbf{w} - 2\mathbf{x} = \mathbf{u} - 3\mathbf{v}$.

2. (5 points) State what it means for a set of vectors $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \ldots, \mathbf{v}_k\}$ to be linearly independent.

3. (10 points) Say if each of the following sets is a vector space. (A "Yes" or "No" answer will suffice.)

   (a) $\{(x, y) \in \mathbb{R}^2 \ : \ x - y = 0\}$

   (b) $\left\{ \begin{bmatrix} x & x+1 \\ x & x \end{bmatrix} \ : \ x \in \mathbb{R} \right\}$

   (c) $\left\{ \begin{bmatrix} 0 & x \\ 2x & 0 \end{bmatrix} \ : \ x \in \mathbb{R} \right\}$

   (d) $\{f \in C(-\infty, \infty) \ : \ f(3) = 0\}$

   (e) $P_9$

   (f) $M_{9,3}$

   (g) $\{(x, y) \in \mathbb{R}^2 \ : \ x \geq 0, y \geq 0\}$

   (h) $\{(x, y) \in \mathbb{R}^2 \ : \ x^2 + y^2 > 0\}$

   (i) $\{(x, y, z) \in \mathbb{R}^3 \ : \ x = 0\}$

   (j) $\{(0, 0, 0)\}$

4. (20 points) The problems on this page concern the matrix $A = \begin{bmatrix} -7 & -5 \\ 10 & 8 \end{bmatrix}$.

(a) Find the eigenvalues of $A$.

(b) For each eigenvalue from part (a), find the corresponding eigenvectors.

5. (10 points) Show that $W = \{(a, b, a + 2b) \ : \ a, b \in \mathbb{R}\}$ is a subspace of $\mathbb{R}^2$.

6. (10 points) Suppose $A$ is a fixed $2 \times 2$ matrix. Show that the set $W = \{X \ : \ AX = XA\}$ is a subspace of $M_{2,2}$.

7. (10 points) Suppose $W$ is the set of all matrices in $M_{2,2}$ that have have determinant equal to 0. Explain why $W$ is not a subspace of $M_{2,2}$.

8. (10 points) Suppose $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are three vectors in a vector space $V$. Without knowing any further information, is it possible to say whether or not the set $\{\mathbf{v} - \mathbf{u}, \ \ \mathbf{w} - \mathbf{v}, \ \ \mathbf{u} - \mathbf{w}\}$ is linearly independent or dependent?

9. (10 points) Is the set $\left\{ \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 2 & 1 \\ 3 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \right\}$ linearly independent or dependent?

10. (10 points) Does the set $\{1 + x,\ \ x + x^2,\ \ x^2 + x^3,\ \ 1 + x^3\}$ span $P_3$?