

Name: _____

R. Hammack

Score: _____

Directions: This is a take-home test. It is due at the beginning of class on Wednesday, February 21. Please answer all questions in the space provided. Consider working the problems on scratch paper, then rewriting them neatly on the test. Additional copies of this test can be downloaded from my web page if needed.

For this test, you may discuss the problems among yourselves and share ideas, but the work you turn in must be your own (not copied). At the end of your solution to each problem, please list who (if anyone) you talked to about that problem, plus any additional information you want me to know (e.g. that you *gave* more help than you *received*, or vice versa, etc.).

- Please clearly indicate all of your row operations (e.g. $R_2 + 3R_4 \rightarrow R_2$, etc).
- Indicate your solution clearly by putting it in a box.
- Constants that are not integers should be expressed as fractions.
- You may consult your text and notes, but **no** other source.
- In order to get full credit, you must show or explain all of your work.

1. (20 points) Find the determinants. If you need more room for (d), reconsider your approach.

$$(a) \begin{vmatrix} -5 & -\frac{1}{2} \\ 10 & 1 \end{vmatrix} =$$

$$(b) \begin{vmatrix} 5 & 1 & 3 \\ -3 & 4 & 4 \\ 5 & 5 & 5 \end{vmatrix} =$$

$$(c) \begin{vmatrix} 1 & -1 & -2 & 5 \\ 3 & 3 & 6 & 1 \\ 0 & 1 & 3 & 0 \\ -2 & 4 & 8 & -2 \end{vmatrix} =$$

$$(d) \begin{vmatrix} 2 & -1 & -2 & 5 & 2 & 1 & 6 \\ 4 & 3 & 4 & 5 & 2 & 1 & 6 \\ 1 & 1 & -3 & 0 & 2 & -1 & 2 \\ 2 & 2 & -6 & 0 & 4 & -2 & 4 \\ 1 & 2 & -6 & 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 1 & 1 & -6 & 1 & 1 & 1 & 1 \end{vmatrix} =$$

2. (10 points) Solve the system
$$\begin{cases} 2w - x - y + 3z = 0 \\ w + x - 2y + z = 1 \\ 2w + x + y - z = 2 \\ 3w - x + y + z = 1 \end{cases}$$

3. (10 points) Solve the system $\begin{cases} 2w + x + y + 3z = 1 \\ 2w - 7x + y + 11z = 1 \\ 2w + 3x + y + z = 1 \end{cases}$

4. (10 points) Solve the system $\begin{cases} x + 4y + 5z = 1 \\ x + 18y + 7z = 2 \\ x - 3y + 4z = 1 \end{cases}$

5. Consider the matrix $A = \begin{bmatrix} 1 & 2 & 1 \\ 3 & 7 & 3 \\ 1 & 2 & 2 \end{bmatrix}$.

(a) (10 points) Find an LU factorization of A .

(b) (10 points) Find the inverse of A . (If it exists.)

6. (6 points) Suppose $A = \begin{bmatrix} 3 & 1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 1 & -2 \end{bmatrix}$. Find X if $A^T B - X = 2I_3$.

7. (6 points) If $D = \begin{bmatrix} -1 & -2 \\ 1 & 0 \end{bmatrix}$, find D^{-2} .

8. (6 points) Suppose A and B are 4×4 matrices, and $\det(A) = 5$, and $\det(B) = -2$. Find $\det(2A^2B^{-3})$.

9. (6 points) Find an example of a 2×2 matrix A for which $A \neq O$, but $A^2 = O$.

10. (6 points) Suppose A is a 7×7 matrix, and $A\mathbf{x} = \mathbf{0}$ for some $\mathbf{x} \neq \mathbf{0}$. Is A invertible or singular? Explain.