Linear Algebra — Test #1 (Chapter 1) — September 1, 2006

Name: ______________________ R. Hammack Score: ________

Directions: This is a take-home test. It is due at the beginning of class on Wednesday, September 6. Please answer all questions in the space provided. Consider working the problems on scratch paper, then rewriting them neatly on the test. Additional copies of this test can be downloaded from my web page if needed.

For this test, you may discuss the problems among yourselves and share ideas, but the work you turn in must be your own (not copied). At the end of your solution to each problem, please list who (if anyone) you talked to about that problem, plus any additional information you want me to know (i.e. that you *gave* more help than you *received*, or vice versa, etc.).

- Use Gaussian elimination or Gauss-Jordan elimination to solve the problems.
- Please clearly indicate all of your row operations (e.g. $R_2 + 3R_4 \to R_2$, etc).
- Put your final matrix in row-echelon or reduced row-echelon form.
- If a system has more than one solution, state the solution set in **parametric form**.
- Indicate your solution clearly by putting it in a $\boxed{\text{box}}$.
- Constants that are not integers should be expressed as fractions.
- You may consult your text and notes, but **no** other source.
- In order to get full credit, you must show all of your work.

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1. (8 points) Solve the system $\left\{\begin{array}{rcrcr} 2x & + & 4y & = & 7 \\ 6x & - & 2y & = & 0 \end{array}\right.$

$$\begin{bmatrix} 2 & 4 & 7 \\ 6 & -2 & 0 \end{bmatrix} \; R_2 - 3R_1 \to R_2 \; \begin{bmatrix} 2 & 4 & 7 \\ 0 & -14 & -21 \end{bmatrix} \; \begin{matrix} \frac{1}{2}R_1 \to R_1 \\ -\frac{1}{14}R_2 \to R_2 \end{matrix} \; \begin{bmatrix} 1 & 2 & \frac{7}{2} \\ 0 & 1 & \frac{3}{2} \end{bmatrix} \; R_1 - 2R_2 \to R_1 \; \begin{bmatrix} 1 & 0 & \frac{1}{2} \\ 0 & 1 & \frac{3}{2} \end{bmatrix}$$

Solution: $\boxed{x = \frac{1}{2}, \quad y = \frac{3}{2}}$

2. (8 points) Solve the system $\left\{\begin{array}{rcrcr} 2x & + & 4y & = & 7 \\ 6x & + & 12y & = & 0 \end{array}\right.$

$$\begin{bmatrix} 2 & 4 & 7 \\ 6 & 12 & 0 \end{bmatrix} \; R_2 - 3R_1 \to R_2 \; \begin{bmatrix} 2 & 4 & 7 \\ 0 & 0 & -21 \end{bmatrix} \; \begin{matrix} \frac{1}{2}R_1 \to R_1 \\ -\frac{1}{21}R_2 \to R_2 \end{matrix} \; \begin{bmatrix} 1 & 2 & \frac{7}{2} \\ 0 & 0 & 1 \end{bmatrix}$$

Second row gives $0 = 1$, so $\boxed{\textbf{no solution}}$.

3. (8 points) Solve the system $\left\{\begin{array}{rcrcr} 2x & + & 4y & = & 7 \\ 6x & + & 12y & = & 21 \end{array}\right.$

$$\begin{bmatrix} 2 & 4 & 7 \\ 6 & 12 & 21 \end{bmatrix} \; R_2 - 3R_1 \to R_2 \; \begin{bmatrix} 2 & 4 & 7 \\ 0 & 0 & 0 \end{bmatrix} \; \frac{1}{2}R_1 \to R_1 \; \begin{bmatrix} 1 & 2 & \frac{7}{2} \\ 0 & 0 & 0 \end{bmatrix}$$

Solution: $\boxed{x = \frac{7}{2} - 2t, \quad y = t}$

4. (12 points) Solve the system $\left\{\begin{array}{rcrcr} 3x & - & 2y & = & 0 \\ x & - & 2y & = & -4 \\ x & - & y & = & -1 \\ 3x & + & 2y & = & 12 \end{array}\right.$

$$\begin{bmatrix} 3 & -2 & 0 \\ 1 & -2 & -4 \\ 1 & -1 & -1 \\ 3 & 2 & 12 \end{bmatrix} \quad R_1 \leftrightarrow R_2 \quad \begin{bmatrix} 1 & -2 & -4 \\ 3 & -2 & 0 \\ 1 & -1 & -1 \\ 3 & 2 & 12 \end{bmatrix} \quad \begin{array}{c} R_2 - 3R_1 \to R_2 \\ R_3 - R_1 \to R_3 \\ R_4 - 3R_1 \to R_4 \end{array} \quad \begin{bmatrix} 1 & -2 & -4 \\ 0 & 4 & 12 \\ 0 & 1 & 3 \\ 0 & 8 & 24 \end{bmatrix} \quad \begin{array}{c} \frac{1}{4}R_2 \to R_2 \\ \\ \frac{1}{8}R_4 \to R_4 \end{array}$$

$$\begin{bmatrix} 1 & -2 & -4 \\ 0 & 1 & 3 \\ 0 & 1 & 3 \\ 0 & 1 & 3 \end{bmatrix} \quad \begin{array}{c} R_3 - R_2 \to R_3 \\ R_4 - R_2 \to R_4 \end{array} \quad \begin{bmatrix} 1 & -2 & -4 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad R_1 + 2R_2 \to R_1 \quad \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Solution: $\boxed{x = 2, \quad y = 3}$

5. (12 points) Solve the system $\left\{\begin{array}{rcrcrcrcr} 5x_1 & - & 10x_2 & - & 5x_3 & + & 15x_4 & = & 25 \\ -3x_1 & + & 6x_2 & + & 2x_3 & + & x_4 & = & 5 \end{array}\right.$

$$\begin{bmatrix} 5 & -10 & -5 & 15 & 25 \\ -3 & 6 & 2 & 1 & 5 \end{bmatrix} \frac{1}{5}R_1 \to R_1 \begin{bmatrix} 1 & -2 & -1 & 3 & 5 \\ -3 & 6 & 2 & 1 & 5 \end{bmatrix} R_2 + 3R_1 \to R_2$$

$$\begin{bmatrix} 1 & -2 & -1 & 3 & 5 \\ 0 & 0 & -1 & 10 & 20 \end{bmatrix} -R_2 \to R_2 \begin{bmatrix} 1 & -2 & -1 & 3 & 5 \\ 0 & 0 & 1 & -10 & -20 \end{bmatrix} R_1 + R_2 \to R_1$$

$$\begin{bmatrix} 1 & -2 & 0 & -7 & -15 \\ 0 & 0 & 1 & -10 & -20 \end{bmatrix}$$

The new system is $\left\{\begin{array}{rcrcrcrcr} x_1 & - & 2x_2 & & & - & 7x_4 & = & -15 \\ & & & & x_3 & - & 10x_4 & = & -20 \end{array}\right.$

So $\left\{\begin{array}{l} x_1 = 2x_2 + 7x_4 - 15 \\ x_3 = 10x_4 - 20 \end{array}\right.$

Solution: $\boxed{x_1 = 2s + 7t - 15 \qquad x_2 = s \qquad x_3 = 10t - 20 \qquad x_4 = t}$

6. (12 points) Solve the system $\begin{cases} 2w - 2x + 2z = 4 \\ w + x - y - z = 2 \\ -w + x - y - z = 2 \\ w - x - y + z = 2 \end{cases}$

$$\left[\begin{array}{rrrrr} 2 & -2 & 0 & 2 & 4 \\ 1 & 1 & -1 & -1 & 2 \\ -1 & 1 & -1 & -1 & 2 \\ 1 & -1 & -1 & 1 & 2 \end{array}\right] \quad \tfrac{1}{2}R_1 \to R_1 \quad \left[\begin{array}{rrrrr} 1 & -1 & 0 & 1 & 2 \\ 1 & 1 & -1 & -1 & 2 \\ -1 & 1 & -1 & -1 & 2 \\ 1 & -1 & -1 & 1 & 2 \end{array}\right] \quad \begin{array}{c} R_2 - R_1 \to R_2 \\ R_3 + R_1 \to R_3 \\ R_4 - R_1 \to R_4 \end{array}$$

$$\left[\begin{array}{rrrrr} 1 & -1 & 0 & 1 & 2 \\ 0 & 2 & -1 & -2 & 0 \\ 0 & 0 & -1 & 0 & 4 \\ 0 & 0 & -1 & 0 & 0 \end{array}\right] \quad R_4 - R_3 \to R_4 \quad \left[\begin{array}{rrrrr} 1 & -1 & 0 & 1 & 2 \\ 0 & 2 & -1 & -2 & 0 \\ 0 & 0 & -1 & 0 & 4 \\ 0 & 0 & 0 & 0 & -4 \end{array}\right] \quad \begin{array}{c} \tfrac{1}{2}R_4 \to R_4 \\ -R_3 \to R_3 \\ -\tfrac{1}{4}R_4 \to R_4 \end{array}$$

$$\left[\begin{array}{rrrrr} 1 & -1 & 0 & 1 & 2 \\ 0 & 1 & -\frac{1}{2} & -1 & 0 \\ 0 & 0 & 1 & 0 & -4 \\ 0 & 0 & 0 & 0 & 1 \end{array}\right]$$

The last row indicates that there are $\boxed{\textbf{No Solutions}}$.

7. (12 points) Solve the system $\begin{cases} -3x + 6y - 9z = 12 \\ x - 2y + 3z = -4 \\ -\frac{1}{3}x + \frac{2}{3}y - z = \frac{4}{3} \end{cases}$

$$\left[\begin{array}{rrrr} -3 & 6 & -9 & 12 \\ 1 & -2 & 3 & -4 \\ -\frac{1}{3} & -\frac{2}{3} & -1 & \frac{4}{3} \end{array}\right] \begin{array}{c} -\frac{1}{3}R_1 \to R_1 \\ -3R_3 \to R_3 \end{array} \left[\begin{array}{rrrr} 1 & -2 & 3 & -4 \\ 1 & -2 & 3 & -4 \\ 1 & -2 & 3 & -4 \end{array}\right] \begin{array}{c} R_2 - R_1 \to R_2 \\ R_3 - R_1 \to R_3 \end{array} \left[\begin{array}{rrrr} 1 & -2 & 3 & -4 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

Solution: $\boxed{x = -4 + 2s - 3t, \quad y = s, \quad z = t}$

8. (12 points) Solve the system $\begin{cases} x + 3y + 2z = 1 \\ 2x + 9y - z = 8 \\ 3x - 6y + 4z = 0 \end{cases}$

$$\left[\begin{array}{rrrr} 1 & 3 & 2 & 1 \\ 2 & 9 & -1 & 8 \\ 3 & -6 & 4 & 0 \end{array}\right] \begin{array}{c} R_2 - 2R_1 \to R_2 \\ R_3 - 3R_1 \to R_3 \end{array} \left[\begin{array}{rrrr} 1 & 3 & 2 & 1 \\ 0 & 3 & -5 & 6 \\ 0 & -15 & -2 & -3 \end{array}\right] R_3 + 5R_2 \to R_3 \left[\begin{array}{rrrr} 1 & 3 & 2 & 1 \\ 0 & 3 & -5 & 6 \\ 0 & 0 & -27 & 27 \end{array}\right] -\tfrac{1}{27}R_3 \to R_3$$

$$\left[\begin{array}{rrrr} 1 & 3 & 2 & 1 \\ 0 & 3 & -5 & 6 \\ 0 & 0 & 1 & -1 \end{array}\right] \begin{array}{c} R_2 + 5R_3 \to R_2 \\ R_1 - 2R_3 \to R_3 \end{array} \left[\begin{array}{rrrr} 1 & 3 & 0 & 3 \\ 0 & 3 & 0 & 1 \\ 0 & 0 & 1 & -1 \end{array}\right] R_1 - R_2 \to R_2 \left[\begin{array}{rrrr} 1 & 0 & 0 & 2 \\ 0 & 3 & 0 & 1 \\ 0 & 0 & 1 & -1 \end{array}\right]$$

$$\tfrac{1}{3}R_2 \to R_2 \left[\begin{array}{rrrr} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & \frac{1}{3} \\ 0 & 0 & 1 & -1 \end{array}\right]$$

Solution: $\boxed{x = 2, \quad y = \frac{1}{3}, \quad z = -1}$

9. (16 points) Solve the system $\left\{\begin{array}{rcrcrcrcrcrcr} x_1 & + & 3x_2 & - & 2x_3 & & & + & 2x_5 & & & = & 0 \\ 2x_1 & + & 6x_2 & - & 5x_3 & - & 2x_4 & + & 4x_5 & - & 3x_6 & = & -1 \\ & & & & 5x_3 & + & 10x_4 & & & + & 15x_6 & = & 5 \\ 2x_1 & + & 6x_2 & & & + & 8x_4 & + & 4x_5 & + & 18x_6 & = & 6 \end{array}\right.$

$$\begin{bmatrix} 1 & 3 & -2 & 0 & 2 & 0 & 0 \\ 2 & 6 & -5 & -2 & 4 & -3 & -1 \\ 0 & 0 & 5 & 10 & 0 & 15 & 5 \\ 2 & 6 & 0 & 8 & 4 & 18 & 6 \end{bmatrix} \begin{array}{c} R_2 - 2R_1 \to R_2 \\ R_4 - 2R_1 \to R_4 \end{array} \begin{bmatrix} 1 & 3 & -2 & 0 & 2 & 0 & 0 \\ 0 & 0 & -1 & -2 & 0 & -3 & -1 \\ 0 & 0 & 5 & 10 & 0 & 15 & 5 \\ 0 & 0 & 4 & 8 & 0 & 18 & 6 \end{bmatrix} -R_2 \to R_2$$

$$\begin{bmatrix} 1 & 3 & -2 & 0 & 2 & 0 & 0 \\ 0 & 0 & 1 & 2 & 0 & 3 & 1 \\ 0 & 0 & 5 & 10 & 0 & 15 & 5 \\ 0 & 0 & 4 & 8 & 0 & 18 & 6 \end{bmatrix} \begin{array}{c} R_3 - 5R_2 \to R_3 \\ R_4 - 4R_2 \to R_4 \end{array} \begin{bmatrix} 1 & 3 & -2 & 0 & 2 & 0 & 0 \\ 0 & 0 & 1 & 2 & 0 & 3 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 6 & 2 \end{bmatrix} \tfrac{1}{6}R_4 \to R_4$$

$$\begin{bmatrix} 1 & 3 & -2 & 0 & 2 & 0 & 0 \\ 0 & 0 & 1 & 2 & 0 & 3 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & \frac{1}{3} \end{bmatrix} R_2 - 3R_4 \to R_2 \begin{bmatrix} 1 & 3 & -2 & 0 & 2 & 0 & 0 \\ 0 & 0 & 1 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & \frac{1}{3} \end{bmatrix} R_1 + 2R_2 \to R_1$$

$$\begin{bmatrix} 1 & 3 & 0 & 4 & 2 & 0 & 0 \\ 0 & 0 & 1 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & \frac{1}{3} \end{bmatrix} R_3 \leftrightarrow R_4 \begin{bmatrix} 1 & 3 & 0 & 4 & 2 & 0 & 0 \\ 0 & 0 & 1 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & \frac{1}{3} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

The new system is $\left\{\begin{array}{rcrcrcrcrcrcr} x_1 & + & 3x_2 & & & + & 4x_4 & + & 2x_5 & & & = & 0 \\ & & & & x_3 & + & 2x_4 & & & & & = & 0 \\ & & & & & & & & & & x_6 & = & \frac{1}{3} \end{array}\right.$

Or $\left\{\begin{array}{l} x_1 = -3x_2 - 4x_4 - 2x_5 \\ x_3 = -2x_4 \\ x_6 = \frac{1}{3} \end{array}\right.$

Solution: $\boxed{x_1 = -3r - 4s - 2t, \quad x_2 = r, \quad x_3 = -2s, \quad x_4 = s, \quad x_5 = t, \quad x_6 = \frac{1}{6}}$