☞ Quiz: Sections 6.1 & 6.2

Linear Algebra
MATH 310
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Name: Key

Score: 10

Directions: Please answer in the space provided. No calculators. Please put all phones, etc., away.

1. Suppose T is a linear transformation defined as $T(x) = Ax$, where $A = \begin{bmatrix} 5 & -3 \\ 1 & 1 \\ 1 & -1 \end{bmatrix}$. $\therefore \mathbb{R}^2 \longrightarrow \mathbb{R}^3$

   (a) State the domain of T. $\mathbb{R}^2$

   (b) State the codomain of T. $\mathbb{R}^3$

   (c) Find the kernel of T.

$$\left[\begin{array}{cc|c} 5 & -3 & 0 \\ 1 & 1 & 0 \\ 1 & -1 & 0 \end{array}\right] \longrightarrow \left[\begin{array}{cc|c} 5 & -3 & 0 \\ 1 & 1 & 0 \\ 0 & -2 & 0 \end{array}\right] \longrightarrow \left[\begin{array}{cc|c} 5 & -3 & 0 \\ 1 & 1 & 0 \\ 0 & 1 & 0 \end{array}\right] \longrightarrow \left[\begin{array}{cc|c} 5 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{array}\right] \longrightarrow \left[\begin{array}{cc|c} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{array}\right] \begin{array}{l} x=0 \\ y=0 \end{array}$$

Kernel is $\left\{ \begin{bmatrix} 0 \\ 0 \end{bmatrix} \right\}$

Comment: ker(T) consists of all solutions of $\begin{bmatrix} 5 & -3 \\ 1 & 1 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ which is $\begin{cases} 5x-3y=0 \\ x+y=0 \\ x-y=0. \end{cases}$ The row reduction above is solving this system

   (d) Find the range of T.

$$\text{Range}(T) = \{A\vec{x} : \vec{x} \in \mathbb{R}^2\} = \left\{ \begin{bmatrix} 5 & -3 \\ 1 & 1 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} : x, y \in \mathbb{R}^2 \right\}$$

$$= \left\{ \begin{bmatrix} 5x-3y \\ x+y \\ x-y \end{bmatrix} : x, y \in \mathbb{R} \right\} = \left\{ x\begin{bmatrix} 5 \\ 1 \\ 1 \end{bmatrix} + y\begin{bmatrix} -3 \\ 1 \\ -1 \end{bmatrix} : x, y \in \mathbb{R}^2 \right\}$$

$$= \boxed{\text{Span}\left(\begin{bmatrix} 5 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} -3 \\ 1 \\ -1 \end{bmatrix} \right)} \longleftarrow \text{Range is this plane in } \mathbb{R}^3. \text{ (2-D)}$$

   (e) nullity(T) = $\dim(\text{kernel}(T)) = \dim(\{\vec{0}\}) = \boxed{0}$

   (f) rank(T) = $\dim(\text{range}(T)) = \boxed{2}$

   (g) Is T one-to-one? Yes because its kernel is $\{\vec{0}\}$.

   (h) Is T onto? No the range is a 2-D plane in $\mathbb{R}^3$, not all of $\mathbb{R}^3$.

2. Suppose $S : P_2 \to P_2$ is a linear transformation for which $S(1) = x - x^2$, $S(x) = 1 + x + 3x^2$ and $S(x^2) = 4$. Find $S(3 - x + 2x^2)$.

$$= S(3) - S(x) + S(2x^2) = S(3 \cdot 1) - S(x) + S(2x^2)$$

$$= 3S(1) - S(x) + 2S(x^2)$$

$$= 3(x - x^2) - (1 + x + 3x^2) + 2 \cdot 4$$

$$= 3x - 3x^2 - 1 - x - 3x^2 + 8 = \boxed{-6x^2 + 2x + 7}$$