☞ Quiz: Section 4.7 | Linear Algebra MATH 310 R. Hammack | November 15, 2018

Name: Key | Score: 10

Directions: Please answer in the space provided. No calculators. Please put all phones, etc., away.

1. Consider the basis $B = \{(2,1,2), (3,4,1), (1,1,1)\}$ of $\mathbb{R}^3$. Find the coordinate vector $[\mathbf{x}]_B$ for $\mathbf{x} = (7,0,11)$.

$[\vec{x}]_B = \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix}$ where $\vec{x} = (7,0,11) = c_1(2,1,2) + c_2(3,4,1) + c_3(1,1,1)$

That is, $(7,0,11) = (2c_1 + 3c_2 + c_3,\ c_1 + 4c_2 + c_3,\ 2c_1 + c_2 + c_3)$

or $\begin{cases} 2c_1 + 3c_2 + c_3 = 7 \\ c_1 + 4c_2 + c_3 = 0 \\ 2c_1 + c_2 + c_3 = 11 \end{cases}$ ← Must solve to find c_1 c_2 c_3

$\left[\begin{array}{ccc|c} 2 & 3 & 1 & 7 \\ 1 & 4 & 1 & 0 \\ 2 & 1 & 1 & 11 \end{array}\right] \xrightarrow[R_3 - R_2 \to R_3]{R_1 - R_2 \to R_1} \left[\begin{array}{ccc|c} 1 & -1 & 0 & 7 \\ 1 & 4 & 1 & 0 \\ 1 & -3 & 0 & 11 \end{array}\right]$ ← this step gets lots of 1's and 0's [good]

$\xrightarrow[R_3 - R_1 \to R_3]{R_2 - R_1 \to R_2} \left[\begin{array}{ccc|c} 1 & -1 & 0 & 7 \\ 0 & 5 & 1 & -7 \\ 0 & -2 & 0 & 4 \end{array}\right] \xrightarrow{-\frac{1}{2}R_3 \to R_3} \left[\begin{array}{ccc|c} 1 & -1 & 0 & 7 \\ 0 & 5 & 1 & -7 \\ 0 & 1 & 0 & -2 \end{array}\right]$

$\xrightarrow[R_2 - 5R_3 \to R_2]{R_1 - R_3 \to R_1} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 5 \\ 0 & 0 & 1 & 3 \\ 0 & 1 & 0 & -2 \end{array}\right] \xrightarrow{R_2 \leftrightarrow R_3} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 5 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 3 \end{array}\right]$

$\boxed{c_1 = 5 \quad c_2 = -2 \quad c_3 = 3}$

$$\boxed{\text{Thus } [\vec{x}]_B = \begin{bmatrix} 5 \\ -2 \\ 3 \end{bmatrix}}$$ ← ANSWER

Check! $5(2,1,2) - 2(3,4,1) + 3(1,1,1) = (7,0,11) = \vec{x}$