Linear Algebra	Quiz for Sections 4.6 and 4.7	November 11, 2006
Name: ______________	R. Hammack	Score: ______

Directions: Please answer all questions in the space provided.
Use of calculators or any form of electronic communication device is strictly forbidden on this quiz.

1. Suppose a matrix A has 45 columns and its rows are linearly independent.
Also, the set $\{\mathbf{x} \in \mathbb{R}^{45} \mid A\mathbf{x} = \mathbf{0}\}$ is 7-dimensional. How many rows does A have? Explain.

The problems states the nullspace $N(A) = \{\mathbf{x} \in \mathbb{R}^{45} \mid A\mathbf{x} = \mathbf{0}\}$ is 7-dimensional, so $\text{nullity}(A) = 7$.
Since the rows of A are linearly independent, then $\text{rank}(A)$ equals the number of rows in A. Then:

$$45 = \text{rank}(A) + \text{nullity}(A)$$

$$45 = (\text{number of rows of } A) + 7$$

It follows that $\boxed{A \text{ has 38 rows.}}$

2. The set $B = \{1 + x + x^2,\ 2 + x + x^2,\ 1 - x + x^2\}$ is a basis for P_2.
Find the coordinate vector $[3 + x^2]_B$.

Note $[3+x^2]_B = \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix}$, where $3 + x^2 = c_1(1 + x + x^2) + c_2(2 + x + x^2) + c_3(1 - x + x^2)$.

Thus we seek numbers c_1, c_2, and c_3 for which $3 + x^2 = (c_1 + 2c_2 + c_3) + (c_1 + c_2 - c_3)x + (c_1 + c_2 + c_3)x^2$.

This gives rise to the following system. $\left\{\begin{array}{rcrcrcr} c_1 & + & 2c_2 & + & c_3 & = & 3 \\ c_1 & + & c_2 & - & c_3 & = & 0 \\ c_1 & + & c_2 & + & c_3 & = & 1 \end{array}\right.$

Solving in the usual way gives

$$\begin{bmatrix} 1 & 2 & 1 & 3 \\ 1 & 1 & -1 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix} \xrightarrow{\substack{R_2 - R_1 \to R_2 \\ R_3 - R_1 \to R_3}} \begin{bmatrix} 1 & 2 & 1 & 3 \\ 0 & -1 & -2 & -3 \\ 0 & -1 & 0 & -2 \end{bmatrix} \xrightarrow{R_2 \leftrightarrow R_3} \begin{bmatrix} 1 & 2 & 1 & 3 \\ 0 & -1 & 0 & -2 \\ 0 & -1 & -2 & -3 \end{bmatrix} \xrightarrow{-R_2 \to R_2}$$

$$\begin{bmatrix} 1 & 2 & 1 & 3 \\ 0 & 1 & 0 & 2 \\ 0 & -1 & -2 & -3 \end{bmatrix} \xrightarrow{\substack{R_1 - 2R_2 \to R_1 \\ R_3 + R_2 \to R_3}} \begin{bmatrix} 1 & 0 & 1 & -1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & -2 & -1 \end{bmatrix} \xrightarrow{-\frac{1}{2}R_3 \to R_3} \begin{bmatrix} 1 & 0 & 1 & -1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 1/2 \end{bmatrix} \xrightarrow{R_1 - R_3 \to R_1}$$

$$\begin{bmatrix} 1 & 0 & 0 & -3/2 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 1/2 \end{bmatrix}$$ We get $c_1 = -3/2,\ \ c_2 = 3$ and $c_3 = 1/2$.

Therefore $\boxed{[3+x^2]_B = \begin{bmatrix} -3/2 \\ 2 \\ 1/2 \end{bmatrix}.}$