☞ Quiz: Section 4.6 | Linear Algebra MATH 310 R. Hammack | November 8, 2018

Name: Key | Score: 10

The following questions concern the matrix $A = \begin{bmatrix} 1 & 4 & 3 & -1 \\ 3 & 12 & 9 & -3 \\ 1 & 4 & 3 & 2 \\ 2 & 8 & 6 & 1 \end{bmatrix}$.

1. Find a basis for the row space of A.

$$\begin{bmatrix} 1 & 4 & 3 & -1 \\ 3 & 12 & 9 & -3 \\ 1 & 4 & 3 & 2 \\ 2 & 8 & 6 & 1 \end{bmatrix} \begin{matrix} R_2 - 3R_1 \to R_2 \\ R_3 - R_1 \to R_3 \\ R_4 - 2R_1 \to R_1 \end{matrix} \begin{bmatrix} 1 & 4 & 3 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 3 \\ 0 & 0 & 0 & 3 \end{bmatrix} \begin{matrix} R_4 - R_3 \to R_4 \\ \frac{1}{3}R_3 \to R_3 \end{matrix} \begin{bmatrix} 1 & 4 & 3 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{matrix} R_1 + R_3 \to R_1 \\ R_2 \leftrightarrow R_3 \end{matrix}$$

$$\begin{bmatrix} 1 & 4 & 3 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$\uparrow \qquad \uparrow$

Basis for row space of A:

$$\boxed{B = \{(1,4,3,0), (0,0,0,1)\}}$$

2. Find a basis for the column space of A.

From the above reduced row echelon form of A, we can see the first and third columns of A span the ~~row~~ column space and they are linearly independent. Therefore

Basis for column space of A is $\boxed{B = \left\{ \begin{bmatrix} 1 \\ 3 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} -1 \\ -3 \\ 2 \\ 1 \end{bmatrix} \right\}}$

3. Find a basis for the null space of A.

Null(A) is the set of solutions to $A\begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$.

To find the solutions, put system in augmented matrix form and row reduce it.

$$\left[\begin{array}{cccc|c} 1 & 4 & 3 & -1 & 0 \\ 3 & 12 & 9 & -3 & 0 \\ 1 & 4 & 3 & 2 & 0 \\ 2 & 8 & 6 & 1 & 0 \end{array}\right] \xrightarrow{\text{(as above)}} \left[\begin{array}{cccc|c} 1 & 4 & 3 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right] \quad \begin{matrix} w = -4x - 3y \\ z = 0 \\ x, y \text{ are free} \end{matrix}$$

$$\text{Null}(A) = \left\{ \begin{bmatrix} -4x-3y \\ x \\ y \\ 0 \end{bmatrix} : x, y \in \mathbb{R} \right\} = \left\{ x\begin{bmatrix} -4 \\ 1 \\ 0 \\ 0 \end{bmatrix} + y\begin{bmatrix} -3 \\ 0 \\ 1 \\ 0 \end{bmatrix} : x, y \in \mathbb{R} \right\}$$

$$= \text{Span}\left(\begin{bmatrix} -4 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -3 \\ 0 \\ 1 \\ 0 \end{bmatrix} \right).$$

Basis for Null Space: $\boxed{B = \left\{ \begin{bmatrix} -4 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -3 \\ 0 \\ 1 \\ 0 \end{bmatrix} \right\}}$