Name: ____________________ R. Hammack Score: ________

Directions: Please answer all questions in the space provided. Use of calculators or any form of electronic communication device is strictly forbidden on this quiz.

Your quiz has one of the following problems:

1. Find a basis for the space of solutions to the following homogeneous linear system.

$$\begin{cases} 3w + 3x + 15y + 15z = 0 \\ w - 3x + y - 3 = 0 \\ 2w + 3x + 11y + 12z = 0 \end{cases}$$

Solving this system in the usual way, we get

$$\begin{bmatrix} 3 & 3 & 15 & 15 & 0 \\ 1 & -3 & 1 & -3 & 0 \\ 2 & 3 & 11 & 12 & 0 \end{bmatrix} \xrightarrow{R_1 \leftrightarrow R_2} \begin{bmatrix} 1 & -3 & 1 & -3 & 0 \\ 3 & 3 & 15 & 15 & 0 \\ 2 & 3 & 11 & 12 & 0 \end{bmatrix} \xrightarrow{\substack{R_2 - 3R_1 \to R_2 \\ R_3 - 2R_1 \to R_3}}$$

$$\begin{bmatrix} 1 & -3 & 1 & -3 & 0 \\ 0 & 12 & 12 & 24 & 0 \\ 0 & 9 & 9 & 18 & 0 \end{bmatrix} \xrightarrow{\substack{\frac{1}{4}R_2 \to R_2 \\ \frac{1}{3}R_3 \to R_3}} \begin{bmatrix} 1 & -3 & 1 & -3 & 0 \\ 0 & 3 & 3 & 6 & 0 \\ 0 & 3 & 3 & 6 & 0 \end{bmatrix} \xrightarrow{\substack{R_1 + R_2 \to R_1 \\ R_3 - R_2 \to R_3}}$$

$$\begin{bmatrix} 1 & 0 & 4 & 3 & 0 \\ 0 & 3 & 3 & 6 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \xrightarrow{\frac{1}{3}R_2 \to R_2} \begin{bmatrix} 1 & 0 & 4 & 3 & 0 \\ 0 & 1 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Now we can read off the solution as $\begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -4y - 3z \\ -y - 2z \\ y \\ z \end{bmatrix} = y \begin{bmatrix} -4 \\ -1 \\ 1 \\ 0 \end{bmatrix} + z \begin{bmatrix} -3 \\ -2 \\ 0 \\ 1 \end{bmatrix}$

Now we can read off the basis as $B = \left\{ \begin{bmatrix} -4 \\ -1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -3 \\ -2 \\ 0 \\ 1 \end{bmatrix} \right\}$

2. Find a basis for the space of solutions to the following homogeneous linear system.

$$\begin{cases} 3w + 3x + 15y + 11z = 0 \\ w - 3x + y + z = 0 \\ 2w + 3x + 11y + 11z = 0 \end{cases}$$

Solving this system in the usual way, we get

$$\begin{bmatrix} 3 & 3 & 15 & 11 & 0 \\ 1 & -3 & 1 & 1 & 0 \\ 2 & 3 & 11 & 11 & 0 \end{bmatrix} \xrightarrow{R_1 \leftrightarrow R_2} \begin{bmatrix} 1 & -3 & 1 & 1 & 0 \\ 3 & 3 & 15 & 11 & 0 \\ 2 & 3 & 11 & 11 & 0 \end{bmatrix} \xrightarrow{\substack{R_2 - 3R_1 \to R_2 \\ R_3 - 2R_1 \to R_3}}$$

$$\begin{bmatrix} 1 & -3 & 1 & -3 & 0 \\ 0 & 12 & 12 & 8 & 0 \\ 0 & 9 & 9 & 9 & 0 \end{bmatrix} \xrightarrow{\substack{\frac{1}{4}R_2 \to R_2 \\ \frac{1}{9}R_3 \to R_3}} \begin{bmatrix} 1 & -3 & 1 & -3 & 0 \\ 0 & 3 & 3 & 2 & 0 \\ 0 & 1 & 1 & 1 & 0 \end{bmatrix} \xrightarrow{\substack{R_1 + 3R_3 \to R_1 \\ R_2 - 3R_3 \to R_2}}$$

$$\begin{bmatrix} 1 & 0 & 4 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 \\ 0 & 1 & 1 & 1 & 0 \end{bmatrix} \xrightarrow{R_3 + R_2 \to R_3} \begin{bmatrix} 1 & 0 & 4 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 \\ 0 & 1 & 1 & 0 & 0 \end{bmatrix} \xrightarrow{\substack{-R_2 \to R_2 \\ R_2 \leftrightarrow R_3}}$$

$$\begin{bmatrix} 1 & 0 & 4 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

Now we can read off the solution as $\begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -4y \\ -y \\ y \\ 0 \end{bmatrix} = y \begin{bmatrix} -4 \\ -1 \\ 1 \\ 0 \end{bmatrix}$

Now we can read off the basis as $B = \left\{ \begin{bmatrix} -4 \\ -1 \\ 1 \\ 0 \end{bmatrix} \right\}$