Linear Algebra | Quiz for Section 4.5 | November 6, 2006

Name: ____________________ R. Hammack Score: ______

Directions: Please answer all questions in the space provided.
Use of calculators or any form of electronic communication device is strictly forbidden on this quiz.

1. For what values of x is the set $B = \{(1,x,x),(x,1,x),(x,x,1)\}$ **not** a basis for $\mathbb{R}^3$?

   To find the x for which B is *not* a basis, we look for the values of x that make B linearly *dependent.* Consider the equation $c_1(1,x,x)+c_2(x,1,x)+c_3(x,x,1)=(0,0,0)$.

   When we put this into the form of a matrix equation, we get: $\begin{bmatrix} 1 & x & x \\ x & 1 & x \\ x & x & 1 \end{bmatrix}\begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

   We know that this equation will have only a trivial solution precisely if the determinant of the coefficient matrix is not zero. Thus B will be linearly independent if the determinant is nonzero, and linearly dependent if it is zero. Thus, to find the values of x for which B is NOT a basis, we look for the values of x that make the determinant zero.

   Now, the coefficient matrix is $A = \begin{bmatrix} 1 & x & x \\ x & 1 & x \\ x & x & 1 \end{bmatrix}$.

   Expanding along the first row, we get

$$\begin{aligned} |A| &= 1(1-x^2) - x(x-x^2) + x(x^2-x) \\ &= (1-x^2) - x^2(1-x) + x^2(x-1) \\ &= (1-x)(1+x) - x^2(1-x) + x^2(x-1) \\ &= (x-1)[-(1+x) + x^2 + x^2] \\ &= (x-1)[2x^2 - x - 1] \\ &= (x-1)(2x+1)(x-1) \end{aligned}$$

   From this we can see that $|A|$ equals 0 when $x = 1$ or $x = -1/2$.
   Therefore $\boxed{B \text{ is not a basis if } x = 1 \text{ or } x = -1/2}$