Linear Algebra	Quiz for Section 4.4	October 26, 2006
Name: ______________________	R. Hammack	Score: ______

Directions: Please answer all questions in the space provided.
Use of calculators or any form of electronic communication device is strictly forbidden on this quiz.

1. Suppose $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are three vectors in a vector space V. Without knowing any further information, is it possible to say whether or not the set $\{\mathbf{v}-\mathbf{u},\ \ \mathbf{w}-\mathbf{v},\ \ \mathbf{u}-\mathbf{w}\}$ is linearly independent or dependent?

   Notice that $1(\mathbf{v}-\mathbf{u})+1(\mathbf{w}-\mathbf{v})+1(\mathbf{u}-\mathbf{w})=\mathbf{0}$, so it follows the set S is $\boxed{\text{linearly dependent.}}$

   Note: One common mistake was to take the equation $c_1(\mathbf{v}-\mathbf{u})+c_2(\mathbf{w}-\mathbf{v})+c_3(\mathbf{u}-\mathbf{w})=\mathbf{0}$, regroup it as $(c_3-c_1)\mathbf{u}+(c_1-c_2)\mathbf{v}+(c_2-c_3)\mathbf{w}=\mathbf{0}$, get $c_3-c_1=0$, $c_1-c_2=0$, $c_2-c_3=0$, and solve the resulting system.

   But there is a problem with this approach. Since we are not given that the vectors $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are linearly independent, we can't deduce that $c_3-c_1=0$, $c_1-c_2=0$, $c_2-c_3=0$.

2. Does the set $S=\{1+x,\ \ x+x^2,\ \ x^2+x^3,\ \ 1+x^3\}$ span P_3?

   Given an arbitrary polynomial $a+bx+cx^2+dx^3$, we want to know if we can always find values for c_1, c_2, c_3, c_4 satisfying $c_1(1+x)+c_2(x+x^2)+c_3(x^2+x^3)+c_4(1+x^3)=a+bx+cx^2+dx^3$.
   Combining, we get $(c_1+c_4)+(c_1+c_2)x+(c_2+c_3)x^2+(c_3+c_4)x^3=a+bx+cx^2+dx^3$, which leads to the following system.

$$\left\{\begin{array}{lllll} c_1 & & & +c_4 & =a \\ c_1 & +c_2 & & & =b \\ & c_2 & +c_3 & & =c \\ & & c_3 & +c_4 & =d \end{array}\right.$$

   Solving:

$$\left[\begin{array}{ccccc} 1 & 0 & 0 & 1 & a \\ 1 & 1 & 0 & 0 & b \\ 0 & 1 & 1 & 0 & c \\ 0 & 0 & 1 & 1 & d \end{array}\right] \begin{array}{c} R_2-R_1\to R_2 \\ \longrightarrow \end{array} \left[\begin{array}{ccccc} 1 & 0 & 0 & 1 & a \\ 0 & 1 & 0 & -1 & b-a \\ 0 & 1 & 1 & 0 & c \\ 0 & 0 & 1 & 1 & d \end{array}\right] \begin{array}{c} R_3-R_2\to R_3 \\ \longrightarrow \end{array} \left[\begin{array}{ccccc} 1 & 0 & 0 & 1 & a \\ 0 & 1 & 0 & -1 & b-a \\ 0 & 0 & 1 & 1 & c-b+a \\ 0 & 0 & 1 & 1 & d \end{array}\right]$$

$$\begin{array}{c} R_4-R_3\to R_4 \\ \longrightarrow \end{array} \left[\begin{array}{ccccc} 1 & 0 & 0 & 1 & a \\ 0 & 1 & 0 & -1 & b-a \\ 0 & 0 & 1 & 1 & c-b+a \\ 0 & 0 & 0 & 0 & d-c+b-a \end{array}\right]$$

   Looking at the last row, you can see the system has no solutions for some values of a, b c and d.
   In particular if $a=0$, $b=0$, $c=0$ and $d=1$, there will be no solution.

   Thus the polynomial $0+0x+0x^2+x^3$ cannot be written as a linear combination of elements in S, so $\boxed{S \text{ does not span } P_3.}$