Quiz: Sections 4.4 & 4.5 | Linear Algebra MATH 310 R. Hammack | November 3, 2016

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Directions: Please answer in the space provided. No calculators. Please put all phones, etc., away.

1. Suppose $S = \{1+2x,\ 1+x+x^2+x^3,\ 1-2x+x^3,\ x+2x^2\}$. Is S a basis for P_3? Why or why not?

Look at the equation

$$a(1+2x)+b(1+x+x^2+x^3)+c(1-2x+x^3)+d(x+x^2) = 0+0x+0x^2+0x^3$$

$$(a+b+c)+(2a+b-2c+d)x+(b+2d)x^2+(b+c)x^3 = 0+0x+0x^2+0x^3$$

This leads to the system

$$\begin{cases} a+b+c = 0 \\ 2a+b-2c+d = 0 \\ b+2d = 0 \\ b+c = 0 \end{cases}$$

Solving:

$$\begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 2 & 1 & -2 & 1 & 0 \\ 0 & 1 & 0 & 2 & 0 \\ 0 & 1 & 1 & 0 & 0 \end{bmatrix} \longrightarrow \begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 0 & -1 & -4 & 1 & 0 \\ 0 & 1 & 0 & 2 & 0 \\ 0 & 1 & 1 & 0 & 0 \end{bmatrix}$$

$$\longrightarrow \begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 0 & -1 & -4 & 1 & 0 \\ 0 & 0 & -4 & 3 & 0 \\ 0 & 0 & -3 & 1 & 0 \end{bmatrix} \longrightarrow \begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 4 & -1 & 0 \\ 0 & 0 & 1 & -3/4 & 0 \\ 0 & 0 & 1 & -1/3 & 0 \end{bmatrix} \longrightarrow \begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 4 & -1 & 0 \\ 0 & 0 & 1 & -3/4 & 0 \\ 0 & 0 & 0 & 5/12 & 0 \end{bmatrix}$$

$$\frac{3}{4}-\frac{1}{3} = \frac{9}{12}-\frac{4}{12} = \frac{5}{12}$$

$$\longrightarrow \begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 4 & -1 & 0 \\ 0 & 0 & 1 & -3/4 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} \longrightarrow \begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 4 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} \longrightarrow \begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

$$\longrightarrow \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

Thus $a=0$, $b=0$, $c=0$, $d=0$

Because the equation has only a trivial solution, the set S is linearly independent.

Now, S is a linearly independent set of 4 elements, and we know P_3 has dimension 4. Thus by the basis test, $\boxed{S \text{ is a basis for } P_3.}$