Linear Algebra | Quiz for Section 4.4 | October 29, 2009

Name: ____________________ R. Hammack Score: ________

Directions: Please answer all questions in the space provided. Use of calculators or any form of electronic communication device is strictly forbidden on this quiz.

Your quiz had ONE of the following two problems.

1. Is the set $\left\{ \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 2 & 1 \\ 3 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \right\}$ linearly independent or dependent?

   We need to see whether the equation $c_1 \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} + c_2 \begin{bmatrix} 2 & 1 \\ 3 & 1 \end{bmatrix} + c_3 \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ has any nontrivial solutions.

   This equation gives rise to the following system.

$$\left\{ \begin{array}{llll} c_1 & +2c_2 & +c_3 & = 0 \\ c_1 & +c_2 & +c_3 & = 0 \\ & 3c_2 & +c_3 & = 0 \\ c_1 & +c_2 & +c_3 & = 0 \end{array} \right.$$

$$\begin{bmatrix} 1 & 2 & 1 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 3 & 1 & 0 \\ 1 & 1 & 1 & 0 \end{bmatrix} \xrightarrow[R_3 - R_1 \to R_3]{R_2 - R_1 \to R_2} \begin{bmatrix} 1 & 2 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 3 & 1 & 0 \\ 0 & -1 & 0 & 0 \end{bmatrix} \xrightarrow[R_4 - R_2 \to R_4]{R_3 + 3R_2 \to R_3} \begin{bmatrix} 1 & 2 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \xrightarrow[\substack{R_1 - R_3 \to R_1 \\ -R_2 \to R_2}]{R_1 + 2R_2 \to R_1} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

   So there's only the trivial solution $c_1 = 0,\ c_2 = 0,\ c_3 = 0$, so the set is **LINEARLY INDEPENDENT.**

2. Is the set $\left\{ \begin{bmatrix} 1 & 1 \\ 2 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 3 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} 2 & -4 \\ 7 & 2 \end{bmatrix} \right\}$ linearly independent or dependent?

   We need to see whether the equation $c_1 \begin{bmatrix} 1 & 1 \\ 2 & 1 \end{bmatrix} + c_2 \begin{bmatrix} 1 & 3 \\ 1 & 1 \end{bmatrix} + c_3 \begin{bmatrix} 2 & -4 \\ 7 & 2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ has any nontrivial solutions.

   This equation gives rise to the following system.

$$\left\{ \begin{array}{llll} c_1 & +c_2 & +2c_3 & = 0 \\ c_1 & +3c_2 & -4c_3 & = 0 \\ 2c_1 & +1c_2 & +7c_3 & = 0 \\ c_1 & +c_2 & +2c_3 & = 0 \end{array} \right.$$

$$\begin{bmatrix} 1 & 1 & 2 & 0 \\ 1 & 3 & -4 & 0 \\ 2 & 1 & 7 & 0 \\ 1 & 1 & 2 & 0 \end{bmatrix} \xrightarrow[\substack{R_3 - 2R_1 \to R_3 \\ R_4 - R_1 \to R_4}]{R_2 - R_1 \to R_2} \begin{bmatrix} 1 & 1 & 2 & 0 \\ 0 & 2 & -6 & 0 \\ 0 & -1 & 3 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \xrightarrow[R_2 + 2R_3 \to R_2]{R_1 + R_3 \to R_1} \begin{bmatrix} 1 & 0 & 5 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & 3 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \xrightarrow[R_2 \leftrightarrow R_2]{-R_3 \to R_3} \begin{bmatrix} 1 & 0 & 5 & 0 \\ 0 & 1 & -3 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

   The solutions are $c_1 = -5t, \quad c_2 = 3t, \quad c_3 = t.$
   Set $t = 1$, and we get a nontrivial solution $c_1 = -5, \quad c_2 = 3, \quad c_3 = 1.$

   Therefore the given set is LINEARLY DEPENDENT.