Linear Algebra | Quiz for Section 4.3 | October 20, 2006

Name: _____ | R. Hammack | Score: _____

Directions: Please answer all questions in the space provided.
Use of calculators or any form of electronic communication device is strictly forbidden on this quiz.

1. Suppose A is a fixed 2×2 matrix. Show that the set $W = \{X \ : \ AX = XA\}$ is a subspace of $M_{2,2}$.

   (a) Suppose that B and C are matrices in the set W.
   This means $AB = BA$ and $AC = CA$.
   Then $A(B + C) = AB + AC = BA + CA = (B + C)A$.
   And $A(B + C) = (B + C)A$ means that $B + C \in W$.
   Therefore W is closed under addition.

   (b) Suppose that $B \in W$ and $c \in \mathbb{R}$.
   The fact that $B \in W$ means $AB = BA$.
   Observe that $A(cB) = c(AB) = c(BA) = (cB)A$.
   And $A(cB) = (cB)A$ means $cB \in W$.
   Therefore W is closed under scalar multiplication.

   Parts (a) and (b) above show that W is closed under addition and scalar multiplication, so W is a subspace.