☞ Quiz: Section 4.3 | Linear Algebra MATH 310 R. Hammack | October 11, 2018

Name: ____________________ Score: ________

1. Let $V = C(-\infty, \infty)$ be the vector space of continuous functions $f : \mathbb{R} \to \mathbb{R}$. Let $W = \{f \in V \ : \ f(0) = 1\}$.
(That is, W is the set of all functions in V that equal 1 when you plug 0 into them.)
Is W is a subspace of V? Why or why not?

The subset $W \subseteq V$ is NOT a subspace of V because W is not closed under scalar multiplication.

To see this let f be the function $f(x) = x^2 + 1$. Notice that $f(0) = 0^2 + 1 = 1$, and therefore $f \in W$.

But $2f$ is the function $2f(x) = 2(x^2 + 1)$, and so $(2f)(0) = 2f(0) = 2(0^2 + 1) = 2 \neq 1$. Hence $2f \notin W$.

In summary, we have found an $f \in W$ for which $2f \notin W$.

Therefore W is not a subspace because it is not closed under scalar multiplication.

2. Let A be a fixed $m \times n$ matrix. Is the set $W = \{\mathbf{x} \in \mathbb{R}^n \ : \ A\mathbf{x} = \mathbf{0}\}$ a subspace of $\mathbb{R}^n$? Why or why not?

First we will show that W is closed under addition.

Suppose $\mathbf{u}, \mathbf{v} \in W$. We wish to show $\mathbf{u} + \mathbf{v} \in W$.

Since $\mathbf{u} \in W$, we know $\boxed{A\mathbf{u} = \mathbf{0}}$, by definition of W. Likewise, since $\mathbf{v} \in W$, we know $\boxed{A\mathbf{v} = \mathbf{0}}$.

Notice that $A(\mathbf{u} + \mathbf{v}) = A\mathbf{u} + A\mathbf{v} = \mathbf{0} + \mathbf{0} = \mathbf{0}$. But $A(\mathbf{u} + \mathbf{v}) = \mathbf{0}$ means $\mathbf{u} + \mathbf{v} \in W$.

Hence W is indeed closed under addition.

Next will show that W is closed under scalar multiplication.

Suppose $c \in \mathbb{R}$ and $\mathbf{u} \in W$. We wish to show $c\mathbf{u} \in W$.

Since $\mathbf{u} \in W$, we know $\boxed{A\mathbf{u} = \mathbf{0}}$, by definition of W.

Notice that $A(c\mathbf{u}) = cA\mathbf{u} = c\mathbf{0} = \mathbf{0}$. But $A(c\mathbf{u}) = \mathbf{0}$ means $c\mathbf{u} \in W$.

Hence W is indeed closed under scalar multiplication.

Thus $\boxed{W \text{ is a subspace of } \mathbb{R}^n \text{ because it is closed under addition and scalar multiplication}}$