☞ Quiz: Sections 4.3 | Linear Algebra MATH 310 R. Hammack | October 13, 2016

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Directions: Please answer in the space provided. No calculators. Please put all phones, etc., away.

1. Recall that $M_{2,2}$ is the set of all 2×2 matrices with entries from $\mathbb{R}$. Recall also that $M_{2,2}$ is a vector space whose addition (and scalar multiplication) is just the regular addition (and scalar multiplication) for matrices.

   Let W be the set of 2×2 matrices whose entries add up to 0. Show that W is a subspace of $M_{2,2}$.

We just need to show (1) W is closed with respect to addition, and (2) W is closed with respect to scalar multiplication.

(1) Suppose $\begin{bmatrix} x & y \\ z & w \end{bmatrix} \in W$ and $\begin{bmatrix} x' & y' \\ z' & w' \end{bmatrix} \in W$

By definition of W this means $\boxed{x+y+z+w=0}$ and $\boxed{x'+y'+z'+w'=0}$

Now look at $\begin{bmatrix} x & y \\ z & w \end{bmatrix} + \begin{bmatrix} x' & y' \\ z' & w' \end{bmatrix} = \begin{bmatrix} x+x' & y+y' \\ z+z' & w+w' \end{bmatrix}$.

The sum of the entries is $x+x'+y+y'+z+z'+w+w'$
$= (x+y+z+w) + (x'+y'+z'+w') = 0+0 = 0$.
Because the entries add up to 0 it follows that the sum $\begin{bmatrix} x & y \\ z & w \end{bmatrix} + \begin{bmatrix} x' & y' \\ z' & w' \end{bmatrix}$ is in W.

(2) Suppose $A = \begin{bmatrix} x & y \\ z & w \end{bmatrix} \in W$ and $c \in \mathbb{R}$. Because $A \in W$ we have $\boxed{x+y+z+w=0}$ by definition of W.
Now look at $cA = \begin{bmatrix} cx & cy \\ cz & cw \end{bmatrix}$. The sum of its entries is $cx+cy+cz+cw = c(x+y+z+w) = c \cdot 0 = 0$
Because the entries add up to 0, it follows that $cA \in W$, so W is closed with respect to scalar multiplication

Therefore W is a subspace of $M_{2,2}$