Linear Algebra | Quiz for Section 4.3 | October 22, 2009

Name: ____________________ R. Hammack Score: ________

Directions: Please answer all questions in the space provided. Use of calculators or any form of electronic communication device is strictly forbidden on this quiz.

1. In this problem $M_{3,3}$ denotes the set of 3×3 matrices and O denotes the 3×3 zero matrix. Also, A is a fixed 3×3 matrix.

   Consider the set $W = \{X \in M_{3,3} \ : \ AX = O\} \subseteq M_{3,3}$.

   Is W a subspace of $M_{3,3}$? Explain why, or why not.

   In what follows, we show that W is a subspace of $M_{3,3}$.

   First, we need to show W is closed under addition.
   Take two arbitrary matrices $Y, Z \in W$.
   Since Y and Z are in W, it follows that $AY = O$ and $AZ = O$.
   We need to show that $Z + Y \in W$.
   Notice that $A(Z + Y) = AZ + AY = O + O = O$.
   Since $A(Z + Y) = O$, it follows that $Z + Y \in W$.

   Next, we need to show W is closed under scalar multiplication.
   Take any $X \in W$ and $c \in \mathbb{R}$.
   We need to show that $cX \in W$.
   Since $X \in W$, we know that $AX = O$.
   Therefore $A(cX) = c(AX) = cO = O$.
   Since $A(cX) = O$, it follows that $cX \in W$.
   Therefore W is closed under scalar multiplication.

   Since W is closed under addition and scalar multiplication,
   it follows by Theorem 4.5 that W is a subspace of $M_{3,3}$.