Linear Algebra	Quiz for Section 3.4	October 7, 2006
Name: ______________	R. Hammack	Score: ______

Directions: Please answer all questions in the space provided. Use of calculators or any form of electronic communication device is strictly forbidden on this quiz.

The following problems concern the matrix $A = \begin{bmatrix} 37 & 105 \\ -14 & -40 \end{bmatrix}$.

1. Find the eigenvalues of A.

$\det(\lambda I - A) = \begin{vmatrix} \lambda - 37 & -105 \\ 14 & \lambda + 40 \end{vmatrix} = (\lambda - 37)(\lambda + 40) + 14 \cdot 105 = \lambda^2 + 3\lambda - 1480 + 1470 = \lambda^2 + 3\lambda - 10 = (\lambda - 2)(\lambda + 5) = 0$

From this you can see that the eigenvalues are $\boxed{\lambda = 2 \text{ and } \lambda = -5.}$

2. For each eigenvalue from Question 1, find the corresponding eigenvectors.

Eigenvectors for $\lambda = 2$:
$(\lambda I - A)\mathbf{x} = \mathbf{0}$
$(2I - A)\mathbf{x} = \mathbf{0}$

$$\begin{bmatrix} -35 & -105 \\ 14 & 42 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -35 & -105 & 0 \\ 14 & 42 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 3 & 0 \\ 1 & 3 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 3 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Solutions: $x = -3y$, i.e. $x = -3t,\ y = t$ so $\boxed{\text{the eigenvectors for } \lambda = 2 \text{ are } \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -3t \\ t \end{bmatrix} = t \begin{bmatrix} -3 \\ 1 \end{bmatrix}}$

Eigenvectors for $\lambda = -5$:
$(\lambda I - A)\mathbf{x} = \mathbf{0}$
$(-5I - A)\mathbf{x} = \mathbf{0}$

$$\begin{bmatrix} -42 & -105 \\ 14 & 35 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -42 & -105 & 0 \\ 14 & 35 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 2 & 5 & 0 \\ 2 & 5 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 2 & 5 & 0 \\ 0 & 0 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 5/2 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Solutions: $x = -\frac{5}{2}y$, i.e. $x = -\frac{5}{2}t,\ y = t$ so $\boxed{\text{the eigenvectors for } \lambda = -5 \text{ are } \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -\frac{5}{2}t \\ t \end{bmatrix} = t \begin{bmatrix} -\frac{5}{2} \\ 1 \end{bmatrix}}$

Note: By scaling the eigenvector $\begin{bmatrix} -\frac{5}{2} \\ 1 \end{bmatrix}$ by a factor of 2, we can say that eigenvectors for $\lambda = -5$ are the scalar multiples of $\begin{bmatrix} -5 \\ 2 \end{bmatrix}$