Linear Algebra | Quiz for Section 3.4 | October 8, 2009

Name: ____________________ R. Hammack Score: ______

Directions: Please answer all questions in the space provided. Use of calculators or any form of electronic communication device is strictly forbidden on this quiz.

The following problems concern the matrix $A = \begin{bmatrix} 2 & 0 \\ -6 & -1 \end{bmatrix}$.

1. Find the eigenvalues of A.

$$\det(\lambda I - A) = \begin{vmatrix} \lambda - 2 & 0 \\ 6 & \lambda + 1 \end{vmatrix} = (\lambda - 2)(\lambda + 1) = 0$$

From this you can see that the eigenvalues are $\boxed{\lambda = 2 \text{ and } \lambda = -1.}$

2. For each eigenvalue from Question 1, find the corresponding eigenvectors.

Eigenvectors for $\lambda = 2$:
$(\lambda I - A)\mathbf{x} = \mathbf{0}$
$(2I - A)\mathbf{x} = \mathbf{0}$

$$\begin{bmatrix} 0 & 0 \\ 6 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 0 \\ 6 & 3 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 1/2 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

We now have the equation: $x = -\frac{1}{2}y$, i.e. $x = -\frac{1}{2}t,\ y = t$ so

$$\boxed{\text{the eigenvectors for } \lambda = 2 \text{ are } \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -\frac{1}{2}t \\ t \end{bmatrix} = t \begin{bmatrix} -\frac{1}{2} \\ 1 \end{bmatrix}}$$

Note: We can scale this by 2 to put the answer into a slightly nicer form:

$$\boxed{\text{the eigenvectors for } \lambda = 2 \text{ are } \begin{bmatrix} x \\ y \end{bmatrix} = t \begin{bmatrix} -1 \\ 2 \end{bmatrix}}$$

Eigenvectors for $\lambda = -1$:
$(\lambda I - A)\mathbf{x} = \mathbf{0}$
$(-I - A)\mathbf{x} = \mathbf{0}$

$$\begin{bmatrix} -3 & 0 \\ 6 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -3 & 0 & 0 \\ 6 & 0 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} -3 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Solutions: $x = 0$, i.e. $y = t$, so $\boxed{\text{the eigenvectors for } \lambda = -1 \text{ are } \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ t \end{bmatrix} = t \begin{bmatrix} 0 \\ 1 \end{bmatrix}}$