

Name: \_\_\_\_\_

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Score: \_\_\_\_\_

**Directions:** Please answer all questions in the space provided. Use of calculators or any form of electronic communication device is strictly forbidden on this quiz.

1. Suppose  $A$  is a square matrix, and  $A^2 = A$ .  
What are the possible values for  $\det(A)$ ? Explain.

$$\begin{aligned}A^2 &= A \\ \det(A^2) &= \det(A) \\ \det(AA) &= \det(A) \\ \det(A)\det(A) &= \det(A) \\ \det(A)^2 - \det(A) &= 0 \\ \det(A)(\det(A) - 1) &= 0\end{aligned}$$

From this equation, it follows that either  $\det(A) = 0$  or  $\det(A) = 1$ .

2. Find all values of  $a$  that make  $\begin{bmatrix} a & a & 0 \\ a^2 & 2 & a \\ 0 & a & a \end{bmatrix}$  singular.

$$\begin{vmatrix} a & a & 0 \\ a^2 & 2 & a \\ 0 & a & a \end{vmatrix} = a \begin{vmatrix} 2 & a \\ a & a \end{vmatrix} - a \begin{vmatrix} a^2 & a \\ 0 & a \end{vmatrix} = a(2a - a^2) - a(a^3 - 0) = 2a^2 - a^3 - a^4 = -a^2(a^2 + a - 2) = a^2(a - 1)(a + 2)$$

From this you can see that the matrix will be singular for  $a = 0$ ,  $a = 1$  and  $a = -2$ .