☞ Quiz: Sections 3.3

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Linear Algebra
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1. Suppose A and B are 4×4 matrices for which $|A| = \frac{1}{2}$ and $|B| = 6$.

(a) $|AB| = |A|\cdot|B| = \frac{1}{2}\cdot 6 = \boxed{3}$

(b) $|A^{-2}| = |(A^{-1})^2| = |A^{-1}A^{-1}| = |A^{-1}|\cdot|A^{-1}| = \frac{1}{|A|}\frac{1}{|A|} = \frac{1}{1/2}\cdot\frac{1}{1/2} = 2\cdot 2 = \boxed{4}$

(c) $|BAB^{-1}| = |B|\cdot|A|\cdot|B^{-1}| = |B||A|\cdot\frac{1}{|B|} = |A| = \boxed{\frac{1}{2}}$

(d) $|-A| = |(-1)A| = (-1)^4|A| = |A| = \boxed{\frac{1}{2}}$

(e) Suppose $C^2 = A^2B^T$. Find $|C|$.

$|C^2| = |A^2B^T|$

$|CC| = |AAB^T|$

$|C|\cdot|C| = |A|\cdot|A|\cdot|B^T|$

$|C|^2 = |A|\cdot|A|\cdot|B|$

$\rightarrow |C|^2 = \frac{1}{2}\frac{1}{2}6$

$|C|^2 = 3/2$

$|C| = \pm\sqrt{\frac{3}{2}}$, that is $|C|$ could be $\sqrt{\frac{3}{2}}$ or $-\sqrt{\frac{3}{2}}$

2. Find all values of x for which the matrix $X = \begin{bmatrix} x-1 & x+4 \\ 3 & x \end{bmatrix}$ is invertible.

It will be invertible provided $|X| \neq 0$.

$|X| = \begin{vmatrix} x-1 & x+4 \\ 3 & x \end{vmatrix} = (x-1)x - 3(x+4) = x^2 - x - 3x - 12$

$= x^2 - 4x - 12 = (x-6)(x+2)$

Thus $|X| = (x-6)(x+2) = 0$ only if $x=6$ or $x=-2$.

Therefore X is invertible for any value of x except $x=-2$ and $x=6$.