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Score: 10

Directions: Please answer in the space provided. No calculators. Please put all phones, etc., away.

1. Suppose  $A$  is a  $5 \times 5$  matrix, and  $A^2 = 2I$ .  
What are the possible values for  $\det(A)$ ? Explain.

$$\det(A^2) = \det(2I)$$

$$\det(A \cdot A) = \det\left(\begin{bmatrix} 2 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 2 \end{bmatrix}\right)$$

$$\det(A) \cdot \det(A) = 2^5$$

$$\det(A)^2 = 32$$

$$\det(A) = \pm \sqrt{32} = \boxed{\pm 4\sqrt{2}}$$

2. Find all values of  $x$  that make  $\begin{bmatrix} 1 & 0 & 3 \\ 2 & -1 & 0 \\ 4 & 2 & x \end{bmatrix}$  singular.

$$\begin{array}{ccc|cc} 1 & 0 & 3 & 1 & 0 \\ 2 & -1 & 0 & 2 & -1 \\ 4 & 2 & x & 4 & 2 \\ \hline & & & x & 0 & 12 \end{array}$$

$-12 \quad 0 \quad 0$

$$\det\left(\begin{bmatrix} 1 & 0 & 3 \\ 2 & -1 & 0 \\ 4 & 2 & x \end{bmatrix}\right) = x + 12 - (-12) = x + 24$$

Therefore the matrix will be singular for  $\boxed{x = -24}$  because that value makes the determinant zero.