

Name: \_\_\_\_\_

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Score: \_\_\_\_\_

**Directions:** Please answer all questions in the space provided.

Use of calculators or any form of electronic communication device is strictly forbidden on this quiz.

1. Suppose  $A, B$  and  $C$  are invertible matrices. Solve the equation  $AXC = CB$  for  $X$ .

$$\begin{aligned}
 AXC &= CB \\
 A^{-1}AXC &= A^{-1}CB \\
 IXC &= A^{-1}CB \\
 XC &= A^{-1}CB \\
 XCC^{-1} &= A^{-1}CBC^{-1} \\
 XI &= A^{-1}CBC^{-1} \\
 X &= A^{-1}CBC^{-1}
 \end{aligned}$$

2. Find the inverse of the matrix  $A = \begin{bmatrix} 3 & 5 & 5 \\ 1 & 2 & 2 \\ 0 & 1 & 2 \end{bmatrix}$ .

$$\begin{array}{ccc}
 \left[ \begin{array}{ccc|ccc} 3 & 5 & 5 & 1 & 0 & 0 \\ 1 & 2 & 2 & 0 & 1 & 0 \\ 0 & 1 & 2 & 0 & 0 & 1 \end{array} \right] & R_1 \leftrightarrow R_2 & \left[ \begin{array}{ccc|ccc} 1 & 2 & 2 & 0 & 1 & 0 \\ 3 & 5 & 5 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 & 0 & 1 \end{array} \right] & R_2 - 3R_1 \rightarrow R_2 \\
 \\
 \left[ \begin{array}{ccc|ccc} 1 & 2 & 2 & 0 & 1 & 0 \\ 0 & -1 & -1 & 1 & -3 & 0 \\ 0 & 1 & 2 & 0 & 0 & 1 \end{array} \right] & \begin{array}{l} R_1 + 2R_2 \rightarrow R_1 \\ R_3 + R_2 \rightarrow R_3 \end{array} & \left[ \begin{array}{ccc|ccc} 1 & 0 & 0 & 2 & -5 & 0 \\ 0 & -1 & -1 & 1 & -3 & 0 \\ 0 & 0 & 1 & 1 & -3 & 1 \end{array} \right] & R_2 + R_3 \rightarrow R_2 \\
 \\
 \left[ \begin{array}{ccc|ccc} 1 & 0 & 0 & 2 & -5 & 0 \\ 0 & -1 & 0 & 2 & -6 & 1 \\ 0 & 0 & 1 & 1 & -3 & 1 \end{array} \right] & -R_2 \rightarrow R_2 & \left[ \begin{array}{ccc|ccc} 1 & 0 & 0 & 2 & -5 & 0 \\ 0 & 1 & 0 & -2 & 6 & -1 \\ 0 & 0 & 1 & 1 & -3 & 1 \end{array} \right]
 \end{array}$$

Thus  $A^{-1} = \boxed{\begin{bmatrix} 2 & -5 & 0 \\ -2 & 6 & -1 \\ 1 & -3 & 1 \end{bmatrix}}$ .