☞ Quiz: Sections 2.3 | Linear Algebra, MATH 310, R. Hammack | September 20, 2016

Name: _____ | Score: 10

Directions: Please answer in the space provided. No calculators. Please put all phones, etc., away.

1. Suppose A, B and X are invertible matrices, and $(5BX)^{-1} = A$. Express X in terms of A and B.

$$\begin{aligned}
(5BX)^{-1} &= A \\
\left((5BX)^{-1}\right)^{-1} &= A^{-1} \\
5BX &= A^{-1} \\
BX &= \frac{1}{5}A^{-1} \\
B^{-1}(BX) &= B^{-1}\left(\frac{1}{5}A^{-1}\right) \\
(B^{-1}B)X &= \frac{1}{5}B^{-1}A^{-1} \\
IX &= \frac{1}{5}B^{-1}A^{-1} \quad \longrightarrow \quad \boxed{X = \tfrac{1}{5}B^{-1}A^{-1}}
\end{aligned}$$

2. Find the inverse of the matrix $A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$, if it exists, or verify that it does not exist.

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{array}\right] \xrightarrow{R_2 - R_1 \to R_2} \left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{array}\right] \xrightarrow{R_2 \leftrightarrow R_3} \left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & -1 & 1 & 0 \end{array}\right] \xrightarrow{R_2 - R_3 \to R_2}$$

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & -1 & 1 & 0 \end{array}\right] \xrightarrow{R_1 - R_2 \to R_1} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & 1 & -1 \\ 0 & 1 & 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & -1 & 1 & 0 \end{array}\right]$$

Answer: $\boxed{A^{-1} = \begin{bmatrix} 0 & 1 & -1 \\ 1 & -1 & 1 \\ -1 & 1 & 0 \end{bmatrix}}$

Check: $AA^{-1} = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}\begin{bmatrix} 0 & 1 & -1 \\ 1 & -1 & 1 \\ -1 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$