Linear Algebra | Quiz for Sections 2.2 and 2.3 | September 17, 2009

Name: ______________ R. Hammack Score: ______

Directions: Please answer all questions in the space provided.
Use of calculators or any form of electronic communication device is strictly forbidden on this quiz.

1. Suppose A, B and C are invertible 4-by-4 matrices, and $AB^{-1}C = I_4$. Express B in terms of A and C.

$$\begin{array}{rcll}
AB^{-1}C & = & I_4 & \\
A^{-1}AB^{-1}C & = & A^{-1}I_4 & \text{(multiply both sides by } A^{-1} \text{ on left)} \\
B^{-1}C & = & A^{-1} & \text{(simplify)} \\
B^{-1}CC^{-1} & = & A^{-1}C^{-1} & \text{(multiply both sides by } C^{-1} \text{ on right)} \\
B^{-1} & = & A^{-1}C^{-1} & \text{(simplify)} \\
(B^{-1})^{-1} & = & (A^{-1}C^{-1})^{-1} & \text{(take the inverse of both sides)} \\
B & = & CA & \text{(simplify)}
\end{array}$$

$\boxed{\textbf{Answer: } B = CA}$

2. Find the inverse of the matrix $A = \begin{bmatrix} 1 & 2 & -1 \\ 3 & 7 & -10 \\ 7 & 16 & -21 \end{bmatrix}$, if it exists, or verify that it does not exist.

$$\left[\begin{array}{rrr|rrr} 1 & 2 & -1 & 1 & 0 & 0 \\ 3 & 7 & -10 & 0 & 1 & 0 \\ 7 & 16 & -21 & 0 & 0 & 1 \end{array}\right] \begin{array}{c} R_2 - 3R_1 \to R_2 \\ R_3 - 7R_1 \to R_3 \\ \longrightarrow \end{array} \left[\begin{array}{rrr|rrr} 1 & 2 & -1 & 1 & 0 & 0 \\ 0 & 1 & -7 & -3 & 1 & 0 \\ 0 & 2 & -14 & -7 & 0 & 1 \end{array}\right] \begin{array}{c} R_3 - 2R_2 \to R_3 \\ \longrightarrow \end{array}$$

$$\left[\begin{array}{rrr|rrr} 1 & 2 & -1 & 1 & 0 & 0 \\ 0 & 1 & -7 & -3 & 1 & 0 \\ 0 & 0 & 0 & -1 & -2 & 1 \end{array}\right] \begin{array}{c} R_1 - 2R_2 \to R_1 \\ -R_3 \to R_3 \\ \longrightarrow \end{array} \left[\begin{array}{rrr|rrr} 1 & 0 & 13 & 7 & -2 & 0 \\ 0 & 1 & -7 & -3 & 1 & 0 \\ 0 & 0 & 0 & 1 & 2 & -1 \end{array}\right] \begin{array}{c} R_1 - 7R_3 \to R_1 \\ R_2 + R_3 \to R_1 \\ \longrightarrow \end{array}$$

$$\left[\begin{array}{rrr|rrr} 1 & 0 & 13 & 0 & -16 & 7 \\ 0 & 1 & -7 & 0 & 7 & -3 \\ 0 & 0 & 0 & 1 & 2 & -1 \end{array}\right]$$

We have now achieved reduced row-echelon form, and the identity does not appear on the left.
Therefore: $\boxed{\textbf{The matrix } A \textbf{ is not invertible.}}$