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Score: 10

Directions: Please answer in the space provided. No calculators. Please put all phones, etc., away.

1. For this problem, $A = \begin{bmatrix} 2 & 3 \\ -2 & -3 \end{bmatrix}$, $B = \begin{bmatrix} 6 & 9 & -3 \\ -4 & -6 & 2 \end{bmatrix}$, $C = \begin{bmatrix} -2 \\ 4 \end{bmatrix}$, and $D = \begin{bmatrix} -2 & 0 \end{bmatrix}$.

Perform the indicated operations or state that they are not possible.

(a) $AB = \begin{bmatrix} 2 & 3 \\ -2 & -3 \end{bmatrix} \begin{bmatrix} 6 & 9 & -3 \\ -4 & -6 & 2 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

(b) $DAC = \begin{bmatrix} -2 & 0 \end{bmatrix} \begin{bmatrix} 2 & 3 \\ -2 & -3 \end{bmatrix} \begin{bmatrix} -2 \\ 4 \end{bmatrix} = \begin{bmatrix} -4 & -6 \end{bmatrix} \begin{bmatrix} -2 \\ 4 \end{bmatrix} = \begin{bmatrix} -16 \end{bmatrix}$

(c) $C - 5D^T = \begin{bmatrix} -2 \\ 4 \end{bmatrix} - 5 \begin{bmatrix} -2 & 0 \end{bmatrix}^T = \begin{bmatrix} -2 \\ 4 \end{bmatrix} - 5 \begin{bmatrix} -2 \\ 0 \end{bmatrix} = \begin{bmatrix} -2 \\ 4 \end{bmatrix} + \begin{bmatrix} 10 \\ 0 \end{bmatrix} = \begin{bmatrix} 8 \\ 4 \end{bmatrix}$

(d) $A^3 = \begin{bmatrix} 2 & 3 \\ -2 & -3 \end{bmatrix} \begin{bmatrix} 2 & 3 \\ -2 & -3 \end{bmatrix} \begin{bmatrix} 2 & 3 \\ -2 & -3 \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ -2 & -3 \end{bmatrix}$

Note:
 $A^3 = A$
(Just a coincidence)

(e) Solve the equation $A + (3X)^T = I_2$ for X .

$$\begin{bmatrix} 2 & 3 \\ -2 & -3 \end{bmatrix} + (3X)^T = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$3X^T = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 2 & 3 \\ -2 & -3 \end{bmatrix}$$

$$3X^T = \begin{bmatrix} -1 & -3 \\ 2 & 4 \end{bmatrix}$$

$$X^T = \frac{1}{3} \begin{bmatrix} -1 & -3 \\ 2 & 4 \end{bmatrix} = \begin{bmatrix} -\frac{1}{3} & -1 \\ \frac{2}{3} & \frac{4}{3} \end{bmatrix}$$

$$X = (X^T)^T = \begin{bmatrix} -\frac{1}{3} & -1 \\ \frac{2}{3} & \frac{4}{3} \end{bmatrix}^T = \begin{bmatrix} -\frac{1}{3} & \frac{2}{3} \\ -1 & \frac{4}{3} \end{bmatrix}$$

Answer:
 $X = \begin{bmatrix} -\frac{1}{3} & \frac{2}{3} \\ -1 & \frac{4}{3} \end{bmatrix}$