Name: _____   R. Hammack   Score: _____

Directions: Please answer in the space provided. Please show all of your work and indicate your row operations. Use of calculators or any form of electronic communication device is strictly forbidden on this quiz.

1. Use either *Gaussian Elimination* or *Gauss-Jordan Elimination* to solve the following system.

$$\begin{cases} x & + & y & - & 5z & = & 3 \\ x & & & - & 2z & = & 1 \\ 2x & - & y & - & z & = & 0 \end{cases}$$

$$\begin{bmatrix} 1 & 1 & -5 & 3 \\ 1 & 0 & -2 & 1 \\ 2 & -1 & -1 & 0 \end{bmatrix} \quad \begin{matrix} R_2 - R_1 \to R_2 \\ R_3 - 2R_1 \to R_3 \\ \longrightarrow \end{matrix} \quad \begin{bmatrix} 1 & 1 & -5 & 3 \\ 0 & -1 & 3 & -2 \\ 0 & -3 & 9 & -6 \end{bmatrix} \quad \begin{matrix} -R_2 \to R_2 \\ \longrightarrow \end{matrix}$$

$$\begin{bmatrix} 1 & 1 & -5 & 3 \\ 0 & 1 & -3 & 2 \\ 0 & -3 & 9 & -6 \end{bmatrix} \quad \begin{matrix} R_3 + 3R_2 \to R_3 \\ \longrightarrow \end{matrix} \quad \begin{bmatrix} 1 & 1 & -5 & 3 \\ 0 & 1 & -3 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad \begin{matrix} R_1 - R_2 \to R_1 \\ \longrightarrow \end{matrix}$$

$$\begin{bmatrix} 1 & 0 & -2 & 1 \\ 0 & 1 & -3 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Now we have achieved reduced row echelon form. This latest matrix corresponds to the following system:

$$\begin{cases} x & & - & 2z & = & 1 \\ & y & - & 3z & = & 2 \\ & & & 0 & = & 0 \end{cases}$$

Therefore:

$$\begin{cases} x & = & 1 + 2z \\ y & = & 2 + 3z \end{cases}$$

SOLUTION: $\boxed{x = 1 + 2t, \quad y = 2 + 3t, \quad z = t, \quad \text{where } t \in \mathbb{R}}$

Too be sure we're right, let's check our work by plugging the values for x,y and z back into the original system:

$$\begin{cases} x & + & y & - & 5z & = & 3 \\ x & & & - & 2z & = & 1 \\ 2x & - & y & - & z & = & 0 \end{cases} \text{ becomes } \begin{cases} (1+2t) & + & (2+3t) & - & 5t & = & 3 \\ (1+2t) & & & - & 2t & = & 1 \\ 2(1+2t) & - & (2+3t) & - & t & = & 0 \end{cases}$$

Notice that once all like terms are canceled the solution is readily seen to make every equation in the system true! Thus we did the work correctly.