| **Midterm** | Linear Algebra<br>MATH 310 | October 18, 2016 |
|---|---|---|
| Name: ______________ | R. Hammack | Score: ______ |

Answer in the space provided. No calculators. Please put all phones, etc., away. Each problem is 10 points.

1. For this problem, $A = \begin{bmatrix} 2 & 3 & -1 \\ 1 & 5 & 5 \end{bmatrix}$, $B = \begin{bmatrix} 2 & -1 \\ -2 & 1 \end{bmatrix}$, $C = \begin{bmatrix} -2 \\ 4 \end{bmatrix}$, and $D = \begin{bmatrix} -2 & 0 \end{bmatrix}$.

   Preform the indicated operations or state that they are not possible.

   (a) $BA =$

   (b) $C - \frac{1}{2}D^T =$

   (c) $B^{-1} =$

   (d) $CD =$

   (e) Solve the equation $X - 3B + 2I_2 = O$ for $X$.

2. Suppose $A, B$ and $C$ are invertible matrices. Solve the equation $AXC = CB$ for $X$.

3. Solve the system $\begin{cases} 4w - 8x - 3y + z = 1 \\ -3w + 6x + 2y + z = 1 \end{cases}$

4. Find the inverse of the matrix $A = \begin{bmatrix} 3 & 5 & 5 \\ 1 & 2 & 2 \\ 0 & 1 & 2 \end{bmatrix}$.

5. A square matrix $A$ is called an *orthogonal matrix* if $AA^T = I$.
If $A$ is orthogonal, what are the possible values for $\det(A)$?

6. Find all values of $k$ that make $\begin{bmatrix} 2-k & 1 \\ 6 & 1-k \end{bmatrix}$ singular.

7. Find $A$, given that $(2A)^{-1} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$.

8. Suppose $\mathbf{u}_1 = (1, 3, 5)$, $\mathbf{u}_2 = (2, -1, 3)$, $\mathbf{u}_3 = (-3, 2, -4)$ and $\mathbf{v} = (-1, 7, 2)$. Is $\mathbf{v}$ a linear combination of $\mathbf{u}_1$, $\mathbf{u}_2$ and $\mathbf{u}_3$?

9. Consider the matrix equation $\begin{bmatrix} 12 & 15 & 5 & 0 \\ -4 & 0 & 0 & 1 \\ 20 & 3 & 1 & 5 \\ 14 & 12 & 4 & 9 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$.

Explain how you know this has more than one solution without making any explicit calculations.

10. Let $A$ be a fixed $2 \times 2$ matrix. Prove that the set $W = \left\{ X \in M_{2,2} \; : \; AX = XA \right\}$ is a subspace of $M_{2,2}$.