Linear Algebra (Math 310) FINAL EXAM December 11, 2006

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1. Suppose $\mathbf{u} = \begin{bmatrix} 1 \\ -2 \\ -2 \end{bmatrix}$, $\mathbf{v} = \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}$, $A = \begin{bmatrix} 2 & -1 & 4 \\ 3 & 1 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 3 \\ -1 & 5 \\ 2 & 1 \end{bmatrix}$.

(a) Find $\mathbf{x}$ if $2\mathbf{u} - 2\mathbf{x} = \mathbf{v}$.

(b) $A\mathbf{u} =$

(c) $AB =$

(d) $BA =$

(e) $A + B^T =$

(f) Give a basis for the column space of A.

(g) $\text{nullity}(A) =$

2. Solve the system: $\begin{cases} w \quad + 2y + z = 1 \\ -2w + x + y - z = -1 \\ w + x + 7y + 3z = 2 \end{cases}$

3. Solve the system: $\begin{cases} x + y + z = 2 \\ x - y - z = 0 \\ x + y - z = -1 \end{cases}$

4. Find the inverse of $A = \begin{bmatrix} 1 & 0 & 2 \\ 1 & 1 & 3 \\ 1 & 0 & 3 \end{bmatrix}$

5. $\begin{vmatrix} 3 & 0 & 5 \\ 1 & 1 & 2 \\ 3 & 3 & 3 \end{vmatrix} =$

6. Suppose linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$ satisfies $T\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} 2 \\ -2 \end{bmatrix}$ and $T\left(\begin{bmatrix} 1 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} 5 \\ 1 \end{bmatrix}$.
Find the standard matrix for T.

7. Consider the matrix $A = \begin{bmatrix} 1 & 0 & 3 & 2 \\ 1 & 2 & 7 & 1 \\ 0 & 1 & 2 & 1 \\ 1 & 1 & 5 & 1 \end{bmatrix}$.

(a) Find a basis for the row space of A.

(b) Find a basis for the nullspace of A.

8. Suppose $T : \mathbb{R}^3 \to \mathbb{R}^2$, is a linear transformation defined as $T\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} y - x \\ x + y + z \end{bmatrix}$.

Suppose $S : \mathbb{R}^2 \to \mathbb{R}^3$, is a linear transformation defined as $S\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} x - y \\ x \\ y \end{bmatrix}$.

Find the standard matrix for $S \circ T$.

9. Suppose a matrix A satisfies $P^{-1}AP = D$, where $P = \begin{bmatrix} 1 & 1 & 1 \\ 4 & 2 & 0 \\ 3 & 1 & 1 \end{bmatrix}$ and $D = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 3 \end{bmatrix}$.
   List the eigenvalues of A, and for each eigenvalue, give a basis for its eigenspace.
   (This can be done without computations.)

10. Decide if the polynomials $\{2 - x,\ \ 2x - x^2,\ \ 6 - 5x + x^2\}$ in P_2 are linearly independent or linearly dependent.

11. Suppose A is an invertible matrix. Prove that if λ is an eigenvalue of A with corresponding eigenvector $\mathbf{x}$, then $\dfrac{1}{\lambda}$ is an eigenvalue of A^{-1} with corresponding eivenvector $\mathbf{x}$.

12. Suppose $B = \{1, x, e^x, xe^x\}$ is the basis for a subspace W of the space of continuous functions $C(-\infty, \infty)$, and $T : W \to W$ is the linear transformation defined as $T(f) = D_x[f]$ (i.e. $T(f)$ equals the derivative of f).

    (a) Find the matrix for T relative to the basis B.

    (b) Find the kernel of T

    (c) Find the rank of T

13. Suppose A is a fixed 2×2 matrix. Prove that the set $W = \{X \in M_{2,2} : XA = AX\}$ is a subspace of $M_{2,2}$.

14. This problem concerns the matrix $A = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$.

(a) Find all eigenvalues for A.

(b) Find all eigenspaces for A.

(c) Is A diagonalizable? Explain.