MATH 310, Section 6.3 Solutions

4. $T(x, y) = (4x + y, 0, 2x - 3y)$ has standard matrix $\begin{bmatrix} 4 & 1 \\ 0 & 0 \\ 2 & -3 \end{bmatrix}$

30. $T_1 : \mathbb{R}^2 \to \mathbb{R}^2$, $T_1(x, y) = (x - 2y, 2x + 3y)$ has standard matrix $\begin{bmatrix} 1 & -2 \\ 2 & 3 \end{bmatrix}$

    $T_2 : \mathbb{R}^2 \to \mathbb{R}^2$, $T_2(x, y) = (y, 0)$has standard matrix $\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$

    The standard matrix for $T_2 \circ T_1$ is $\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & -2 \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ 0 & 0 \end{bmatrix}$

    The standard matrix for $T_1 \circ T_2$ is $\begin{bmatrix} 1 & -2 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 2 \end{bmatrix}$

36. $T(x_1, x_2, x_3) = (x_1, x_1 + x_2, x_1 + x_2 + x_3)$ has standard matrix $A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$

    You can see that $\det(A) = 1$, so A is invertible. Now find its inverse.

    $$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 \end{array}\right] \to \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & -1 & 1 & 0 \\ 0 & 1 & 1 & -1 & 0 & 1 \end{array}\right] \to \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & -1 & 1 & 0 \\ 0 & 0 & 1 & 0 & -1 & 1 \end{array}\right]$$

    Thus $A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}$, and this is the standard matrix for T^{-1}.

    Consequently, $T^{-1}\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\right) = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} x_1 \\ -x_1 + x_2 \\ -x_2 + x_3 \end{bmatrix}$

    Therefore $\boxed{T^{-1}(x_1,\ x_2,\ x_3) = (x_1,\ x_2 - x_1,\ x_3 - x_2)}$

38. $T(x, y) = (x + y, 3x + 3y)$ has standard matrix $A = \begin{bmatrix} 1 & 1 \\ 3 & 3 \end{bmatrix}$.

    Since $|A| = 0$, this matrix is not invertible, so $\boxed{T \text{ is not invertible.}}$

50. Let $T : P_2 \to P_4$ be given by $T(p) = x^2 p$. Find the matrix for T relative to the bases $B = \{1, x, x^2\}$ and $B' = \{1, x, x^2, x^3, x^4\}$

    $$A = [\ [T(1)]_{B'}\ \ [T(x)]_{B'}\ \ [T(x^2)]_{B'}\ ] = [\ [x^2]_{B'}\ \ [x^3]_{B'}\ \ [x^4]_{B'}\ ] = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

    Checking, $A[a+bx+cx^2]_B = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ a \\ b \\ c \end{bmatrix} = [ax^2+bx^3+cx^4]_{B'} = [T(a+bx+cx^2)]_{B'}$.