MATH 310, Section 4.7 Solutions

6. Suppose $B = \{(4,0,7,3),(0,5,-1,-1),(-3,4,2,1),(0,1,5,0)\}$ and $[\mathbf{x}]_B = [-2,3,4,1]^T$.

Since $[\mathbf{x}]_B = [-2,3,4,1]^T = \begin{bmatrix} -2 \\ 3 \\ 4 \\ 1 \end{bmatrix}$, it follows

$\mathbf{x} = -2(4,0,7,3) + 3(0,5,-1,-1) + 4(-3,4,2,1) + 1(0,1,5,0) = (-20,32,-4,-5)$.

Given that the standard basis for $\mathbb{R}^4$ is $S = \{(1,0,0,0),(0,1,0,0),(0,0,1,0),(0,0,0,1)\}$, we have

$$\boxed{[\mathbf{x}]_S = [-20,32,-4,-5]^T = \begin{bmatrix} -20 \\ 32 \\ -4 \\ -5 \end{bmatrix}.}$$

8. Find the coordinate of $\mathbf{x} = (-26,32)$ relative to the basis $B = \{(-6,7),(4,-3)\}$.

This involves finding a solution to $x(-6,7) + y(4,-3) = (-26,32)$, which gives rise to the system:

$$\begin{cases} -6x + 4y &= -26 \\ 7x - 3y &= 32 \end{cases}$$

$$\begin{bmatrix} -6 & 4 & -26 \\ 7 & -3 & 32 \end{bmatrix} \to \begin{bmatrix} -6 & 4 & -26 \\ 1 & 1 & 6 \end{bmatrix} \to \begin{bmatrix} 1 & 1 & 6 \\ -6 & 4 & -26 \end{bmatrix} \to \begin{bmatrix} 1 & 1 & 6 \\ 0 & 10 & 10 \end{bmatrix} \to \begin{bmatrix} 1 & 1 & 6 \\ 0 & 1 & 1 \end{bmatrix} \to \begin{bmatrix} 1 & 0 & 5 \\ 0 & 1 & 1 \end{bmatrix}$$

Thus $5(-6,7) + 1(4,-3) = (-26,32)$, so $\boxed{[\mathbf{x}]_B = \begin{bmatrix} 5 \\ 1 \end{bmatrix}.}$

14. Find the transition matrix from $B = \{(1,0),(0,1)\}$ to $B' = \{(1,1),(5,6)\}$.

Setting up the problem as in Theorem 4.21, we get

$$\left[\begin{array}{cc|cc} 1 & 5 & 1 & 0 \\ 1 & 6 & 0 & 1 \end{array}\right] \to \left[\begin{array}{cc|cc} 1 & 5 & 1 & 0 \\ 0 & 1 & -1 & 1 \end{array}\right] \to \left[\begin{array}{cc|cc} 1 & 0 & 6 & -5 \\ 0 & 1 & -1 & 1 \end{array}\right],$$ so $\boxed{\text{the transition matrix is } P^{-1} = \begin{bmatrix} 6 & -5 \\ -1 & 1 \end{bmatrix}}$

30. Find the coordinate matrix of $p = 3x^2 + 114x + 13$ relative to the standard basis $S = \{1, x, x^2\}$ of P_2.

Since $p = 13 \cdot 1 + 114 \cdot x + 3 \cdot x^2$, it follows that $\boxed{[p]_S = \begin{bmatrix} 13 \\ 114 \\ 3 \end{bmatrix}.}$

36. Suppose P is the transition matrix from B'' to B' and Q is the transition matrix from B' to B. What is the transition matrix from B to B''?

Since P is the transition matrix from B'' to B', we have $P[\mathbf{x}]_{B''} = [\mathbf{x}]_{B'}$, so

$$[\mathbf{x}]_{B''} = P^{-1}[\mathbf{x}]_{B'}. \tag{1}$$

Since Q is the transition matrix from B' to B, we have $Q[\mathbf{x}]_{B'} = [\mathbf{x}]_B$, so

$$[\mathbf{x}]_{B'} = Q^{-1}[\mathbf{x}]_B. \tag{2}$$

Taking equation (1) and replacing the $[\mathbf{x}]_{B'}$ with $Q^{-1}[\mathbf{x}]_B$ (By equation (2)) we get

$$[\mathbf{x}]_{B''} = P^{-1}Q^{-1}[\mathbf{x}]_B. \tag{3}$$

From this, $(P^{-1}Q^{-1})[\mathbf{x}]_B = [\mathbf{x}]_{B''}$, which means $\boxed{P^{-1}Q^{-1} = (QP)^{-1} \text{ is the transition matrix from } B \text{ to } B''.}$