**MATH 310, Section 3.3 Solutions**

20. Suppose $A$ and $B$ are $3 \times 3$ matrices with $|A| = 10$ and $|B| = 12$.

    (a) $|AB| = |A||B| = (10)(12) = \boxed{120}$

    (b) $|A^4| = |AAAA| = |A||A||A||A| = (10)^4 = \boxed{10000}$

    (c) $|2A| = 2^3|A| = 8(12) = \boxed{96}$ (Remember: $A$ has 3 rows.)

    (d) $|(AB)^T| = |AB| = |A||B| = (10)(12) = \boxed{120}$

    (e) $|A^{-1}| = \dfrac{1}{|A|} = \boxed{\dfrac{1}{10}}$

28. $A = \begin{bmatrix} 2 & -1/2 & 8 \\ 1 & -1/4 & 4 \\ -5/2 & 3/2 & 8 \end{bmatrix}$. Notice that the first row is twice the second row, so the determinant is zero. Hence the matrix is $\boxed{\text{NOT INVERTIBLE.}}$

32. Consider the system $\begin{cases} x_1 + x_2 - x_3 = 4 \\ 2x_1 - x_2 + x_3 = 6 \\ 3x_1 - 2x_2 + 2x_3 = 0 \end{cases}$ which is $\begin{bmatrix} 1 & 1 & -1 \\ 2 & -1 & 1 \\ 3 & -2 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 6 \\ 6 \\ 0 \end{bmatrix}$.

    Since the third column of the coefficient matrix is the negative of the second column, we can tell the determinant of the coefficient matrix is zero. Then, by condition 2 of the Equivalent Conditions For a Nonsingular Matrix (page 144), $\boxed{\text{the system does not have a unique solution.}}$

36. Find the values of $k$ for which $\begin{bmatrix} k-1 & 2 \\ 2 & k+2 \end{bmatrix}$ is singular.
    The determinant is $(k-1)(k+2) - 4 = k^2 + k - 2 - 4 = k^2 + k - 6 = (k+3)(k-2)$.
    This is zero when $k = -3$ or $k = 2$.
    Thus the answer is that $\boxed{\text{the matrix is singular if } k = -3 \text{ or } k = 2.}$

54. Suppose $A$ and $B$ are $3 \times 3$ matrices with $|A| = 4$ and $|B| = 5$.

    (a) $|AB| = |A||B| = (4)(5) = \boxed{20}$

    (b) $|2A| = 2^3|A| = \boxed{32}$

    (c) $A$ and $B$ are both non-singular because their determinants are non-zero.

    (d) $|A^{-1}| = \dfrac{1}{|A|} = \boxed{\dfrac{1}{4}}$

    $|B^{-1}| = \dfrac{1}{|B|} = \boxed{\dfrac{1}{5}}$

    (e) $|(AB)^T| = |AB| = |A||B| = (4)(5) = \boxed{20}$